EE 435

Lecture 27

Data Converter Characterization

- Linearity Metrics
- Spectral Characterizaton

INL-based ENOB

Consider initially the continuous INL definition for an ADC where the INL of an ideal ADC is $X_{LSB}/2$

Assume
$$INL = \theta X_{REF} = \upsilon X_{LSBR}$$

where X_{LSBR} is the LSB based upon the defined resolution

Define the effective LSB by
$$\mathcal{X}_{LSBEFF} = \frac{\mathcal{X}_{REF}}{2^{n_{EQ}}}$$

Thus
$$INL = \theta 2^{n_{EQ}} X_{LSBEFF}$$

Since an ideal ADC has an INL of $X_{LSB}/2$, express INL in terms of ideal ADC
$$INL = \left[\theta 2^{(n_{EQ}+1)}\right]\left(\frac{X_{LSBEFF}}{2}\right)$$

Setting term in [ ] to 1, can solve for n_{EQ} to obtain

$$ENOB = n_{EQ} = \log_2\left(\frac{1}{2\theta}\right) = n_R - 1 - \log_2(\upsilon)$$

where n_R is the defined resolution

Review From Last Lecture

INL-based ENOB

$$ENOB = n_R - 1 - \log_2(\upsilon)$$

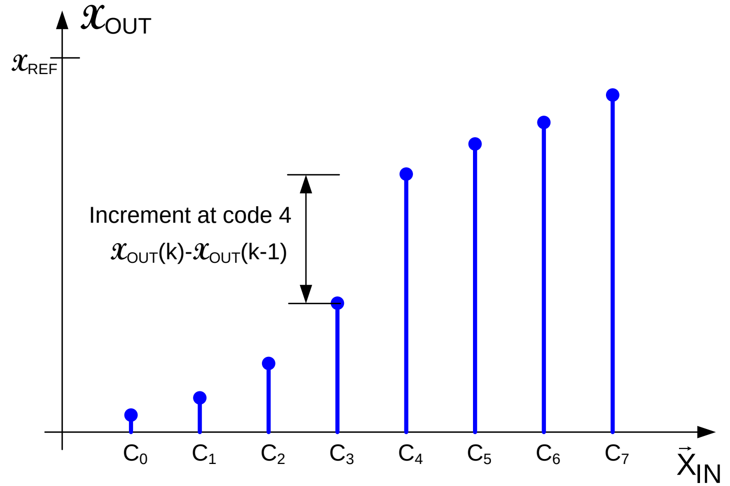
Consider an ADC with specified resolution of n_R and INL of v LSB

υ	ENOB
½	n
1	n-1
2	n-2
4	n-3
8	n-4
16	n-5

Review From Last Lecture

Differential Nonlinearity (DAC)

Nonideal DAC



Increment at code k is a signed quantity and will be negative if $X_{OUT}(k)<X_{OUT}(k-1)$

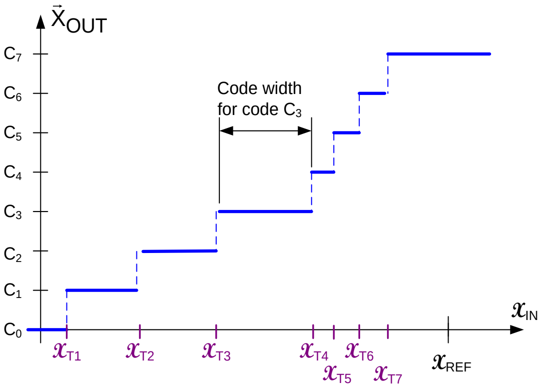
$$DNL(k)=\frac{X_{OUT}(k)-X_{OUT}(k-1)-X_{LSB}}{X_{LSB}}$$

$$DNL=\max_{1\le k\le N-1}\{|DNL(k)|\}$$

DNL=0 for an ideal DAC

Differential Nonlinearity (ADC)

Nonideal ADC



$$\mathrm{DNL}(k)=\frac{x_{T(k+1)}-x_{Tk}-x_{LSB}}{x_{LSB}}$$

$$\mathrm{DNL}=\max_{2\le k\le N-1}\left\{\left|\mathrm{DNL}(k)\right|\right\}$$

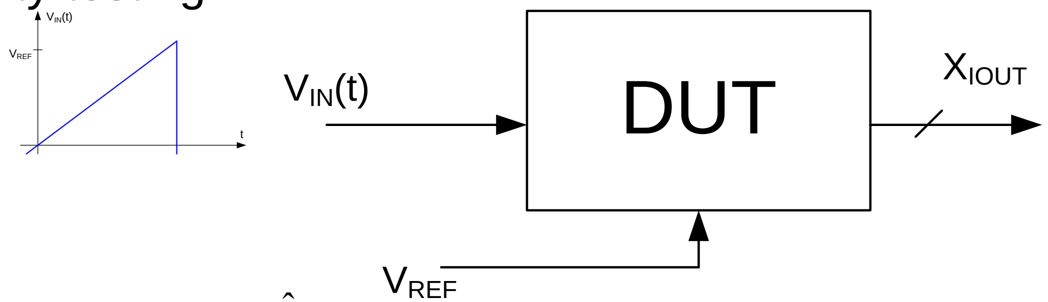
DNL=0 for an ideal ADC

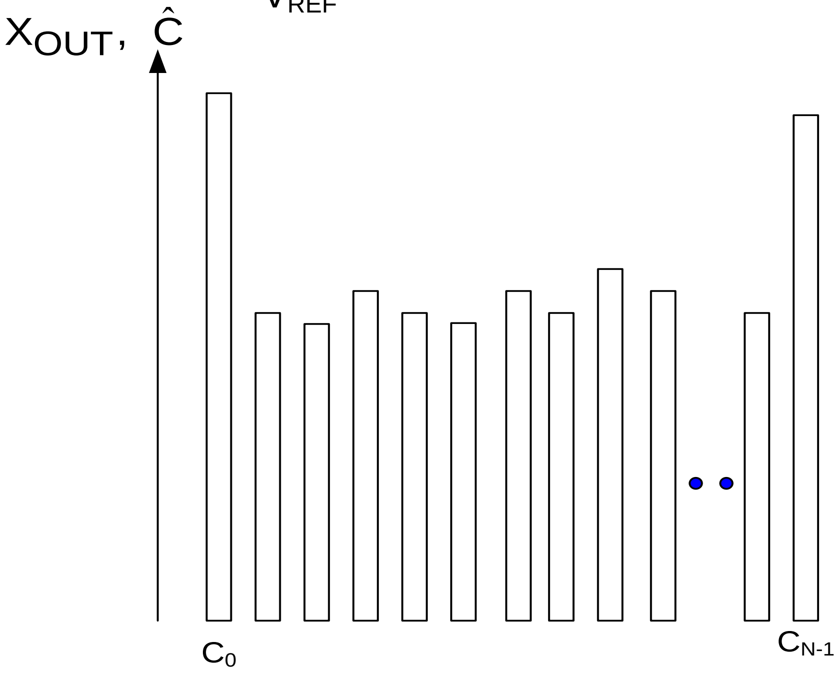
Note: In some nonideal ADCs, two or more break points could cause transitions to the same code C_k making the definition of DNL ambiguous

Review From Last Lecture

Linearity Measurements (testing)

Code density testing:

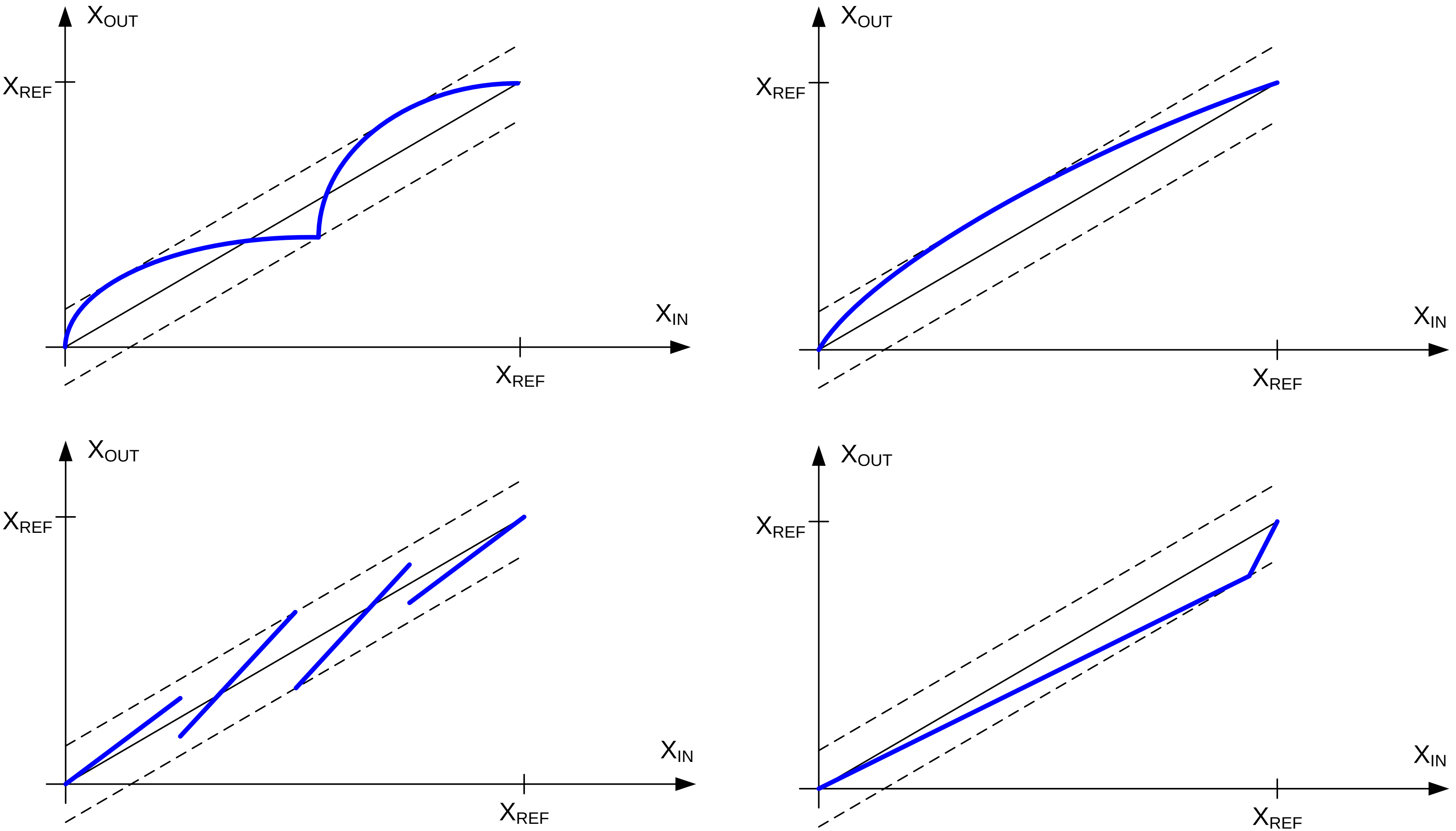




- First and last bins generally have many extra counts (and thus no useful information)
- Typically average 16 or 32 hits per code

INL Often Not a Good Measure of Linearity

Four identical INL with dramatically different linearity



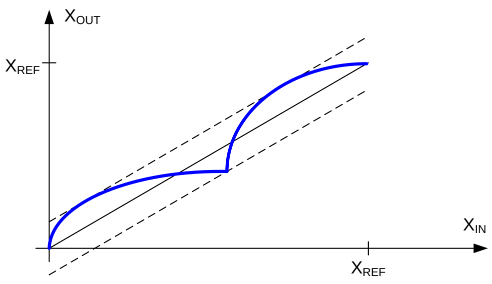
Linearity Issues

- INL is often not adequate for predicting the linearity performance of a data converter
- Distortion (or lack thereof) is of major concern in many applications
- Distortion is generally characterized in terms of the harmonics that may appear in a waveform

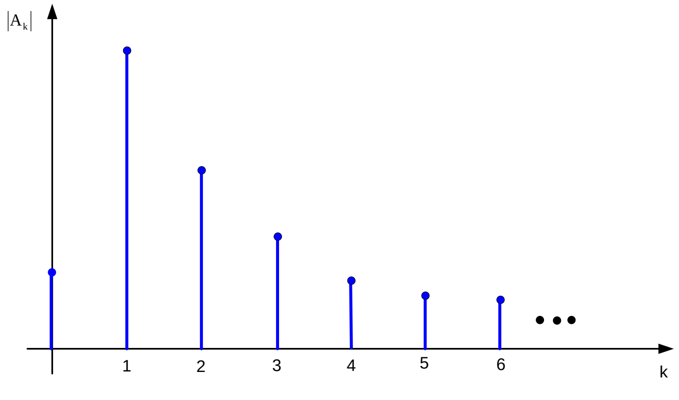
Review From Last Lecture

Two Popular Methods of Linearity Characterization

- Integral and Differential *Nonlinearity* *(metrics: INL,* DNL)

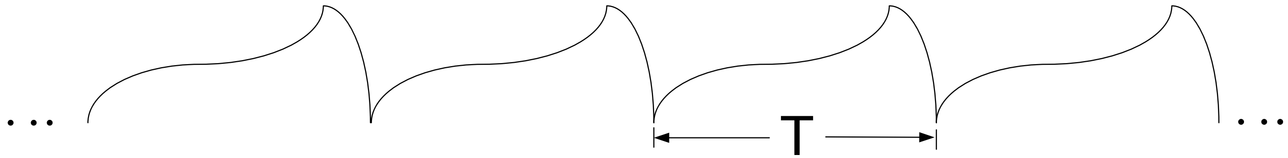


- Spectral Characterization (Based upon spectral harmonics of sinusoidal signals   metrics: THD, SFDR, SDR SNR)



Review From Last Lecture

Spectral Analysis



If f(t) is periodic

$$f(t) = A_0 + \sum_{k=1}^{\infty} A_k \sin(k\omega t + \theta_k)$$

alternately

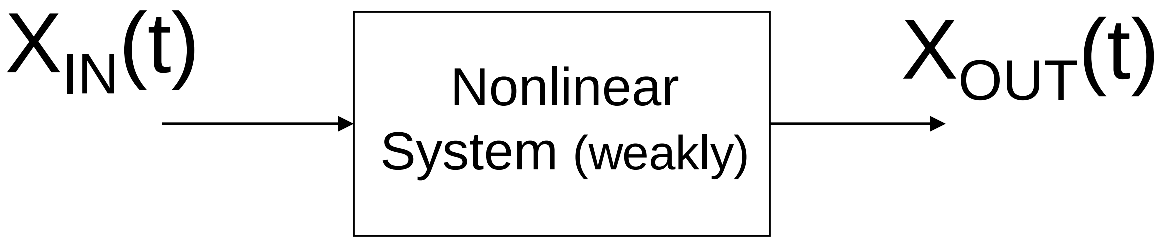
$$f(t) = A_0 + \sum_{k=1}^{\infty} a_k \sin(k\omega t) + \sum_{k=1}^{\infty} b_k \cos(k\omega t) \qquad \omega = \frac{2\pi}{T}$$

$$A_k = \sqrt{a_k^2 + b_k^2}$$

Termed the Fourier Series Representation of f(t)

Review From Last Lecture

Spectral Analysis



Distortion Types:

Frequency Distortion

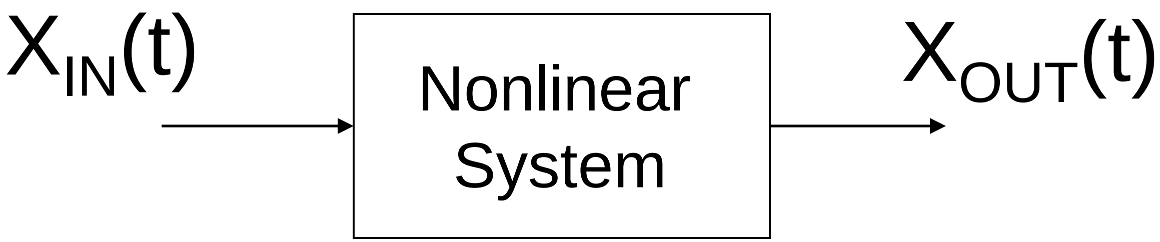
Nonlinear Distortion (alt. harmonic distortion)

Frequency Distortion: Amplitude and phase of system is altered but output is linearly related to input

Nonlinear Distortion: System is not linear, frequency components usually appear in the output that are not present in the input

Spectral Analysis is the characterization of a system with a periodic input with the Fourier series relationships between the input and output waveforms

Spectral Analysis



If $X_{IN}(t) = X_m \sin(\omega t + \theta)$

$$X_{OUT}(t) = A_0 + \sum_{k=1}^{\infty} A_k \sin(k\omega t + \theta_k)$$

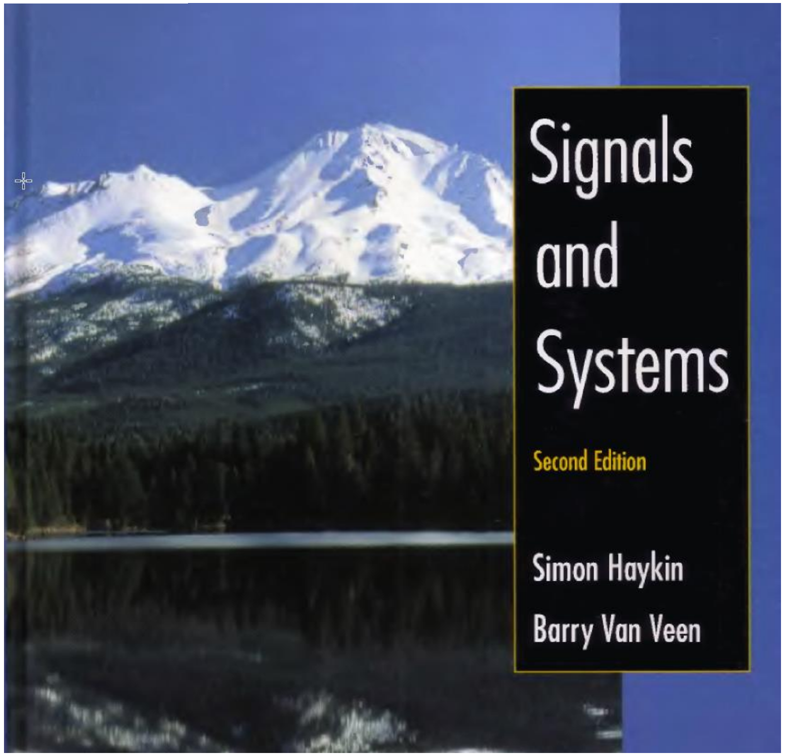
All spectral performance metrics depend upon the sequences $\langle A_k \rangle_{k=0}^{\infty}$ $\langle \theta_k \rangle_{k=1}^{\infty}$

Spectral performance metrics of interest: SNDR, SDR, THD, SFDR, IMOD

Alternately

$$X_{OUT}(t) = A_0 + \sum_{k=1}^{\infty} a_k \sin(k\omega t) + \sum_{k=1}^{\infty} b_k \cos(k\omega t) \qquad A_k = \sqrt{a_k^2 + b_k^2} \qquad \theta_k = \tan^{-1}\left(\frac{b_k}{a_k}\right)$$

3.3 *Fourier Representations for Four Classes of Signals*



There are four distinct Fourier representations, each applicable to a different class of signals. The four classes are defined by the periodicity properties of a signal and whether the signal is continuous or discrete in time. The Fourier series (FS) applies to continuous-time periodic signals, and the discrete-time Fourier series (DTFS) applies to discrete-time periodic signals. Nonperiodic signals have Fourier transform representations. The Fourier transform (FT) applies to a signal that is continuous in time and nonperiodic. The discrete-time Fourier transform (DTFT) applies to a signal that is discrete in time and nonperiodic. Table 3.1 illustrates the relationship between the temporal properties of a signal and the appropriate Fourier representation.

FS, FT, DTFS,DTFT

DFT (**Discrete Fourier Transform**) is a practical version of the **DTFT**, that is computed for a finite-length discrete signal. The **DFT** becomes equal to the **DTFT** as the length of the sample becomes infinite and the **DTFT** converges to the continuous Fourier transform **in the** limit of the sampling frequency going to infinity. Oct 27, 2014

The DFT is the most important discrete transform, used to perform Fourier analysis in many practical applications.[1] In digital signal processing, the

DFS, DTFT, and DFT

Herein we describe the relationship between the Discrete Fourier Series (DFS), Discrete Time Fourier Transform (DTFT), and the Discrete Fourier Transform (DFT). Why? The real reason is that the DFT is easily implemented on a computer and is part of every mathematics package, so it would be nice to know how to determine or approximate the DFT and DTFT on a computer.

Fast Fourier transform - Wikipedia

A **fast Fourier transform** (**FFT**) is an algorithm that computes the discrete Fourier transform (DFT) of a sequence, or its inverse (IDFT). Fourier analysis converts a signal from its original domain (often time or space) to a representation in the frequency domain and vice versa.

DFT,DFS,FFT,IDFT

The “Fourier” Representations:

FS, FT, DTFS,DTFT

DFT, DFS, FFT, IDFT

Really fundamental concepts but varying notation and maybe varying perceptions

Spectral Characterization

Assume f(t) is periodic with period T and band-limited

f(t) is sampled N times at with sampling interval T_S $\qquad NT_s=T$

time domain $\qquad f(t)=\sum_{i=1}^{N} A_k \sin(k\omega t+\theta_k)$ $\qquad$ 2N parameters

$$\vec{x} =< f(T_S), f(2T_S), \ldots . f(NT_S) >$$

(A_k, θ_k)

IDFT

DFT

frequency domain $\qquad \vec{X} =< X_1, X_2,,,, X_N >$ $\qquad$ 2N parameters

(X_k are complex)

f(t)=IDFT (DFT(f(t)))

Spectral Characterization

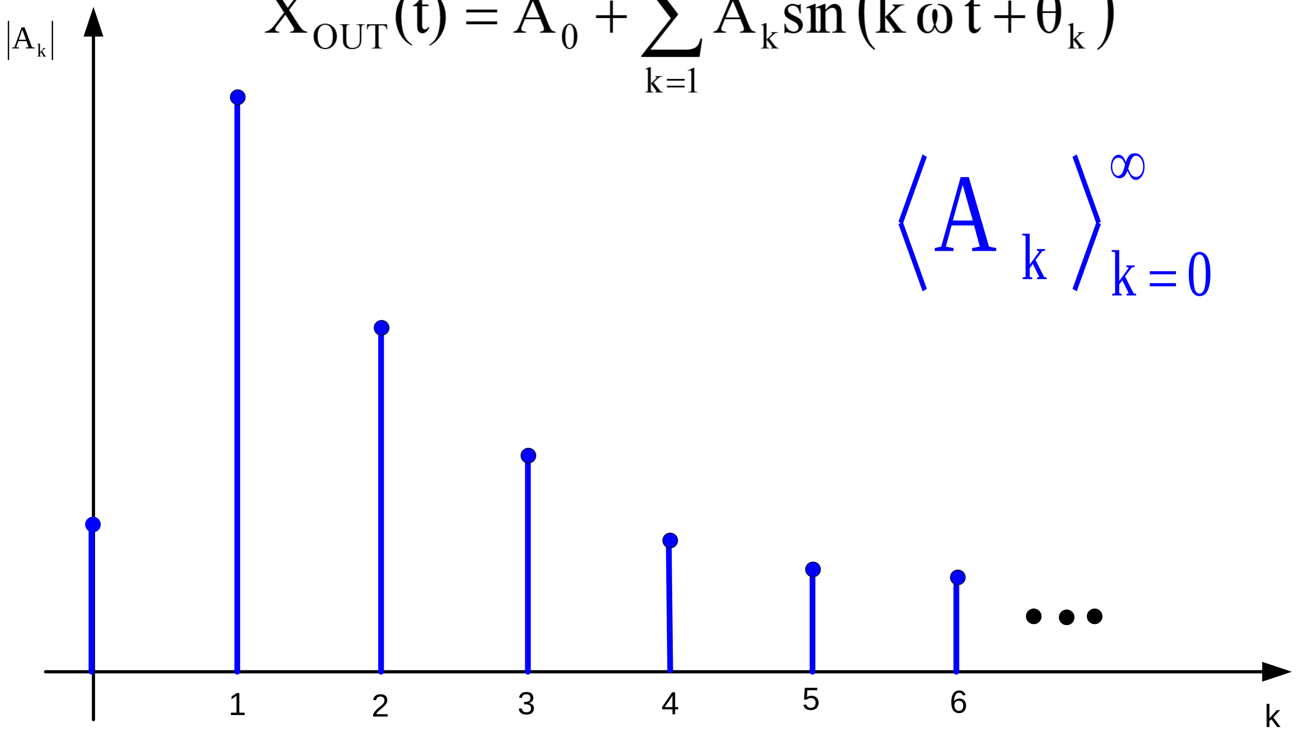
Will focus on how Fourier Series Representation of a periodic signal is altered when it passes through a weakly nonlinear system

Relationship between DFT and continuous-time Fourier Series representation is fundamental to characterizing spectral performance of a weakly nonlinear system

Distortion Analysis

$$X_{OUT}(t) = A_0 + \sum_{k=1}^{\infty} A_k \sin\left(k\,\omega\,t + \theta_k\right)$$

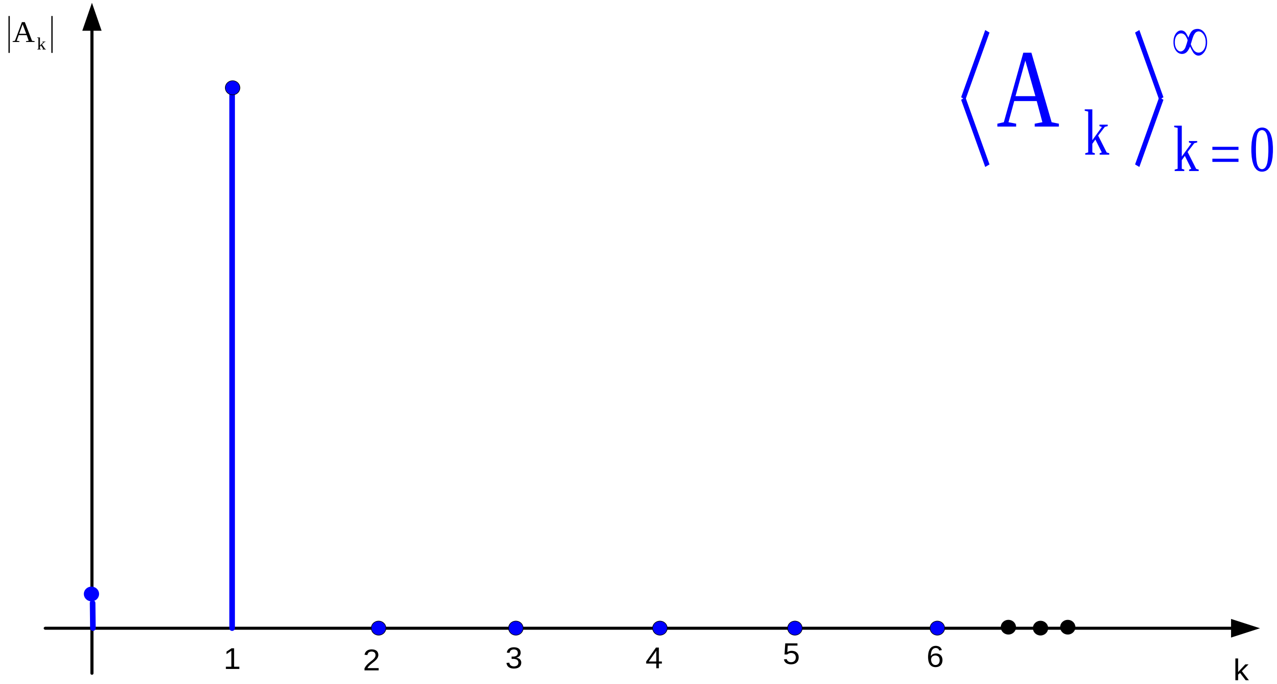
$$\left\langle A_k \right\rangle_{k=0}^{\infty}$$



- Often termed the DFT coefficients (will show later)
- Spectral lines, not a continuous function

A_1 is termed the fundamental
A_k is termed the kth harmonic

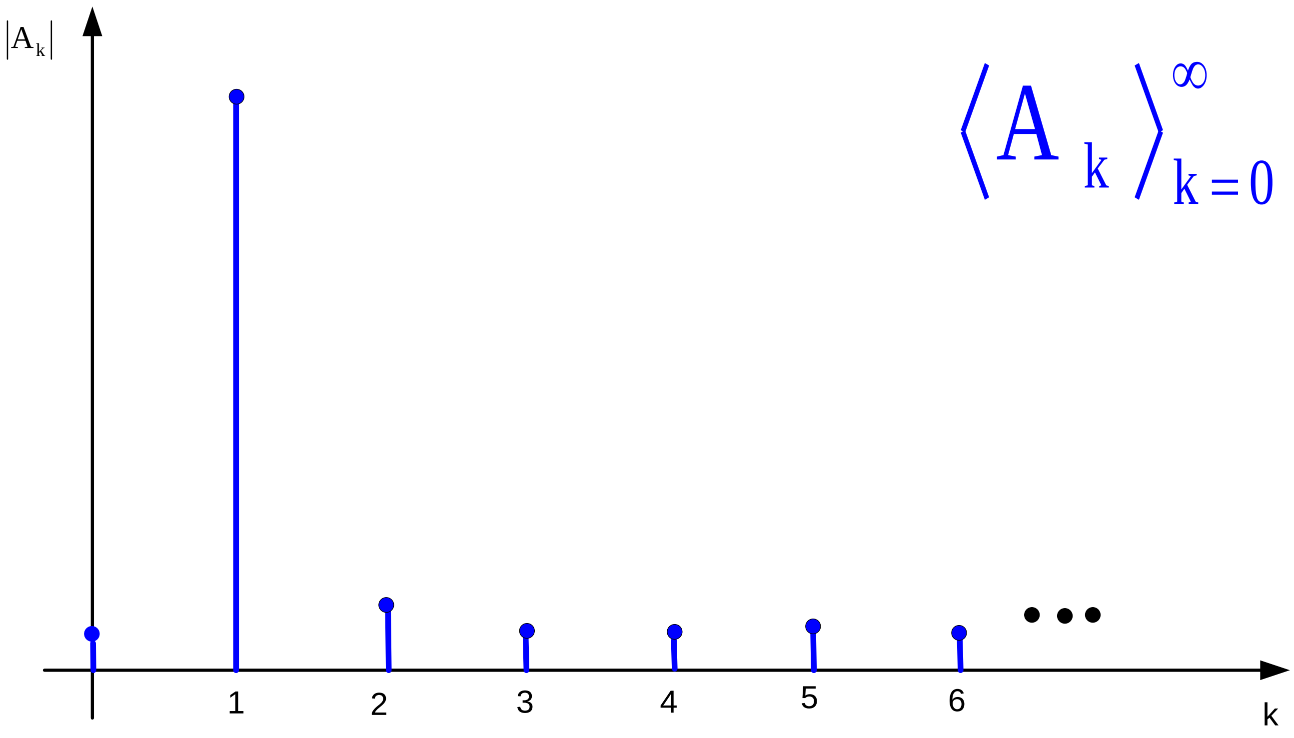
Distortion Analysis



$$\langle A_k \rangle_{k=0}^{\infty}$$

Often <u>ideal</u> response will have only fundamental present and all remaining spectral terms will vanish

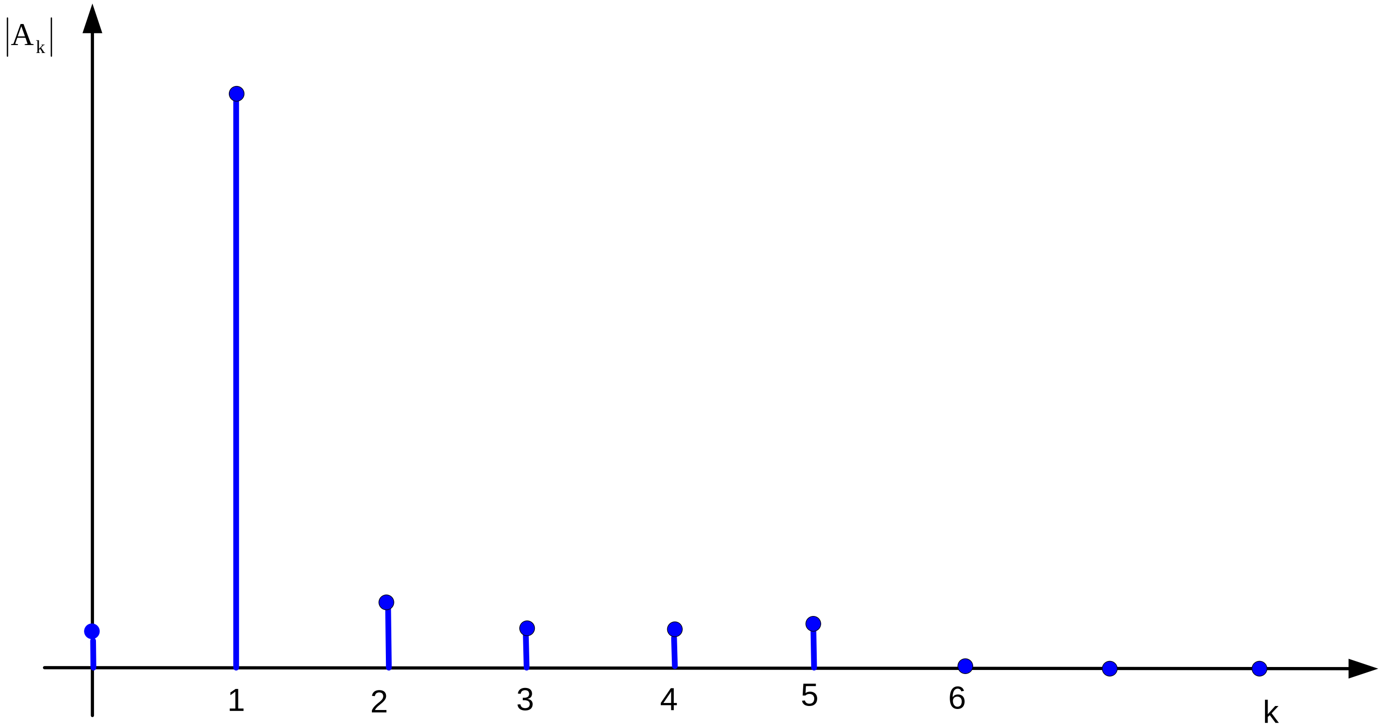
Distortion Analysis



For a low distortion signal, the 2nd and higher harmonics are generally much smaller than the fundamental

The magnitude of the harmonics generally decrease rapidly with k for low distortion signals

Distortion Analysis



Assume f(t) is periodic with period $T = \frac{1}{f}$

f(t) is band-limited to frequency 2π f k_X if A_k=0 for all k>k_x

Where $\langle A_k \rangle_{k=0}^{\infty}$ are the Fourier series coefficients of f(t)

Distortion Analysis

Total Harmonic Distortion, THD

$$\mathrm{THD} = \frac{\text{RMS voltage in harmonics}}{\text{RMS voltage of fundamenta l}}$$

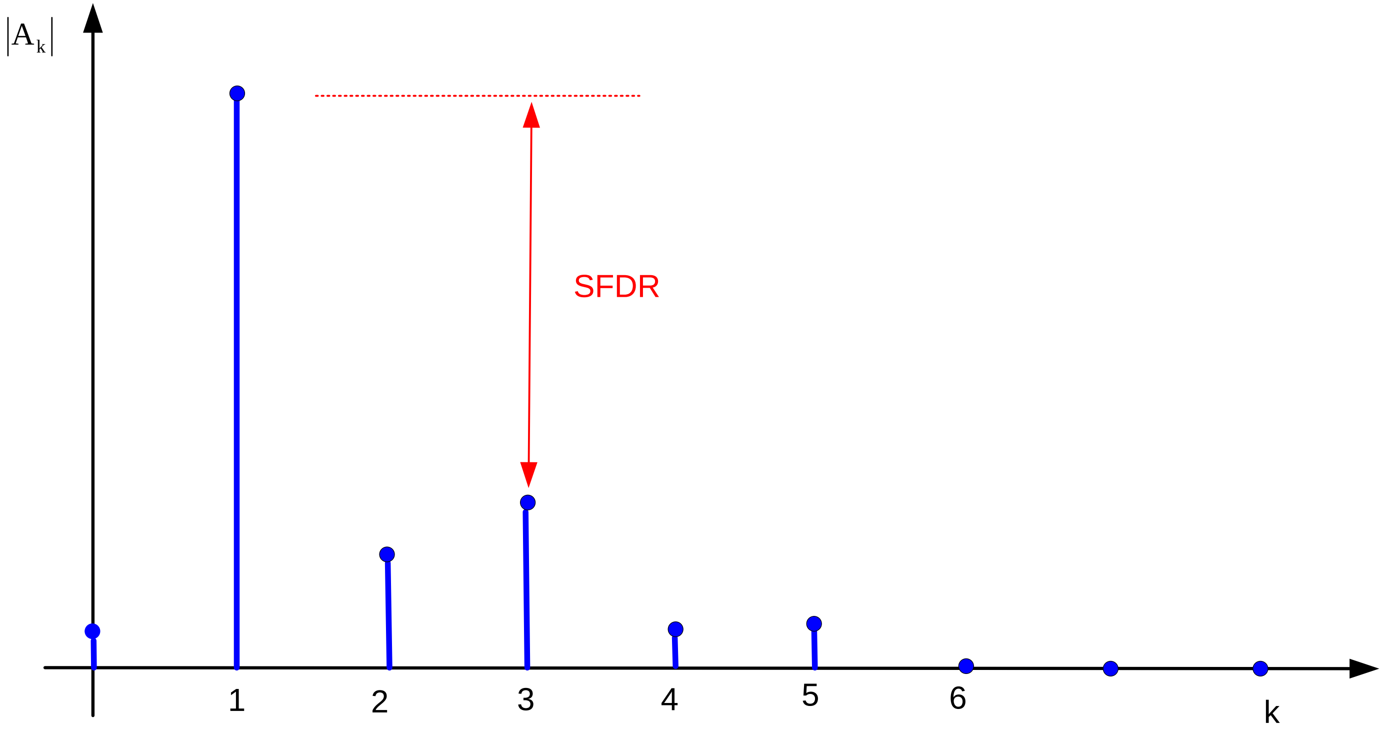
$$\mathrm{THD} = \frac{\sqrt{\left(\frac{A_2}{\sqrt{2}}\right)^2 + \left(\frac{A_3}{\sqrt{2}}\right)^2 + \left(\frac{A_4}{\sqrt{2}}\right)^2 + \ldots}}{\frac{A_1}{\sqrt{2}}}$$

$$\mathrm{THD} = \frac{\sqrt{\sum_{k=2}^{\infty} A_k^2}}{A_1}$$

Distortion Analysis

Spurious Free Dynamic Range, SFDR

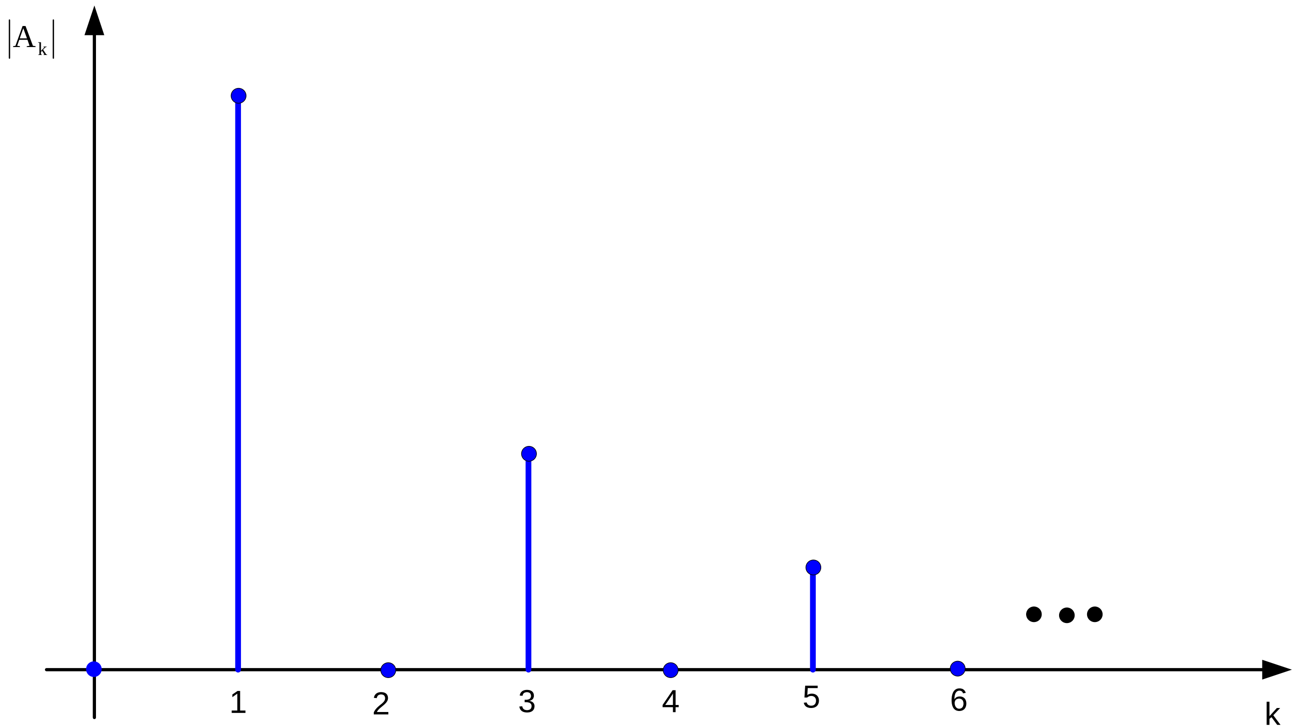
The SFDR is the difference between the fundamental and the largest harmonic



SFDR is usually determined by either the second or third harmonic

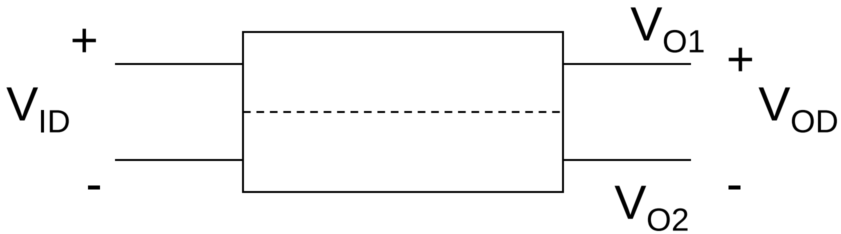
Distortion Analysis

In a fully differential symmetric circuit, all even harmonics are absent in the differential output !



Distortion Analysis

Theorem: In a fully differential symmetric circuit, all even-order terms are absent in the Taylor's series output for symmetric differential excitations !



Proof: Expanding in a Taylor's series around V_{ID}=0, we obtain

$$V_{01} = f\left(V_{ID}\right) = \sum_{k=0}^{\infty} h_k \left(V_{ID}\right)^k$$

$$V_{02} = f\left(-V_{ID}\right) = \sum_{k=0}^{\infty} h_k \left(-V_{ID}\right)^k$$

$$V_{OD} = V_{01} - V_{02} = \sum_{k=0}^{\infty} h_k \left(V_{ID}\right)^k - \sum_{k=0}^{\infty} h_k \left(-V_{ID}\right)^k$$

$$V_{OD} = \sum_{k=0}^{\infty} h_k \left[\left(V_{ID}\right)^k - \left(-V_{ID}\right)^k\right]$$

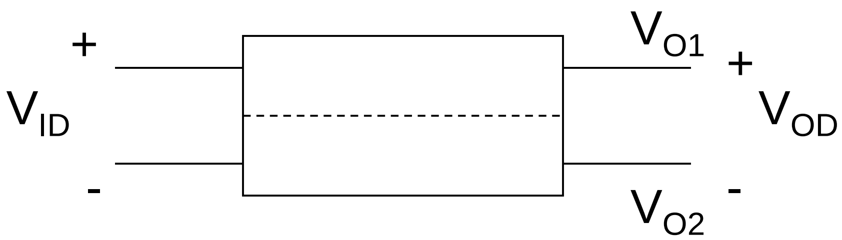
$$V_{OD} = \sum_{k=0}^{\infty} h_k \left[\left(V_{ID}\right)^k - \left(-1\right)^k \left(V_{ID}\right)^k\right]$$

When k is even, term in [ ] vanishes

Distortion Analysis

Theorem: In a fully differential symmetric circuit, all even harmonics are absent in the differential output for symmetric differential excitations !

Proof:



Recall:

$$\sin^n(x) = \begin{cases} \displaystyle\sum_{k=0}^{\frac{n-1}{2}} h_k \sin\left((n-2k)x\right) & \textit{for n odd} \\ \displaystyle\sum_{k=0}^{\frac{n-2}{2}} g_k \sin\left((n-2k)x + \theta_k\right) & \textit{for n even} \end{cases}$$

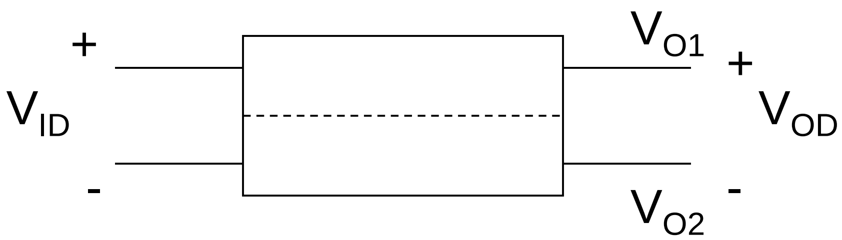
where h_k, g_k, and θ_k are constants

That is, odd powers of $\sin^n(x)$ have only odd harmonics present and even powers have only even harmonics present

Distortion Analysis

Theorem: In a fully differential symmetric circuit, all even harmonics are absent in the differential output for symmetric differential sinusoidal excitations !

Proof:



Expanding in a Taylor's series around V_{ID}=0, we obtain

$$V_{O1} = f\left(V_{ID}\right) = \sum_{k=0}^{\infty} h_k V_{ID}^k \quad \text{and} \quad V_{O2} = f\left(-V_{ID}\right) = \sum_{k=0}^{\infty} h_k (-V_{ID})^k$$

Assume V_{ID}=Ksin(ωt) W.L.O.G. assume K=1

$$V_{O1} = \sum_{k=0}^{\infty} h_k \left[\sin\left(\omega t\right)\right]^k \qquad V_{O2} = \sum_{k=0}^{\infty} h_k \left[-\sin\left(\omega t\right)\right]^k$$

$$V_{OD} = V_{O1} - V_{O2} = \sum_{k=0}^{\infty} h_k \left(\left[\sin\left(\omega t\right)\right]^k - \left[-\sin\left(\omega t\right)\right]^k\right) = \sum_{k=0}^{\infty} h_k \left(\left[\sin\left(\omega t\right)\right]^k - \left(-1\right)^k \left[\sin\left(\omega t\right)\right]^k\right)$$

Observe the even-ordered powers and hence even harmonics are absent in this last sum

Distortion Analysis

How are spectral components determined?

By integral

$$A_k = \frac{1}{\omega T}\left(\int_{t_1}^{t_1+T} f(t)e^{-jk\omega t}dt + \int_{t_1}^{t_1+T} f(t)e^{jk\omega t}dt\right)$$

or

$$a_k = \frac{2}{\omega T}\int_{t_1}^{t_1+T} f(t)\sin(kt\omega)dt \qquad b_k = \frac{2}{\omega T}\int_{t_1}^{t_1+T} f(t)\cos(kt\omega)dt$$

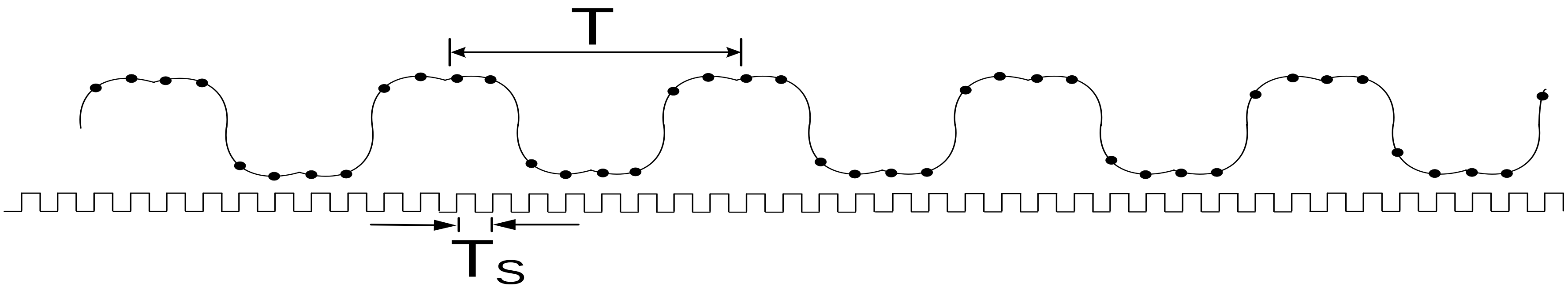
Integral is very time consuming, particularly if large number of components are required

By DFT (with some restrictions that will be discussed)

By FFT (special computational method for obtaining DFT)

Distortion Analysis

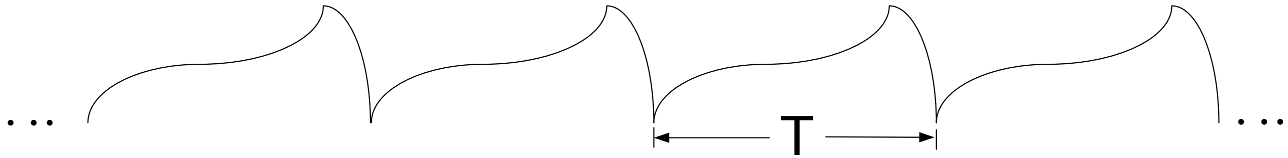
How are spectral components determined?



Consider sampling f(t) at uniformly spaced points in time T_S seconds apart

This gives a sequence of samples $\left\langle f\left(kT_S\right)\right\rangle_{k=1}^{N}$

Distortion Analysis



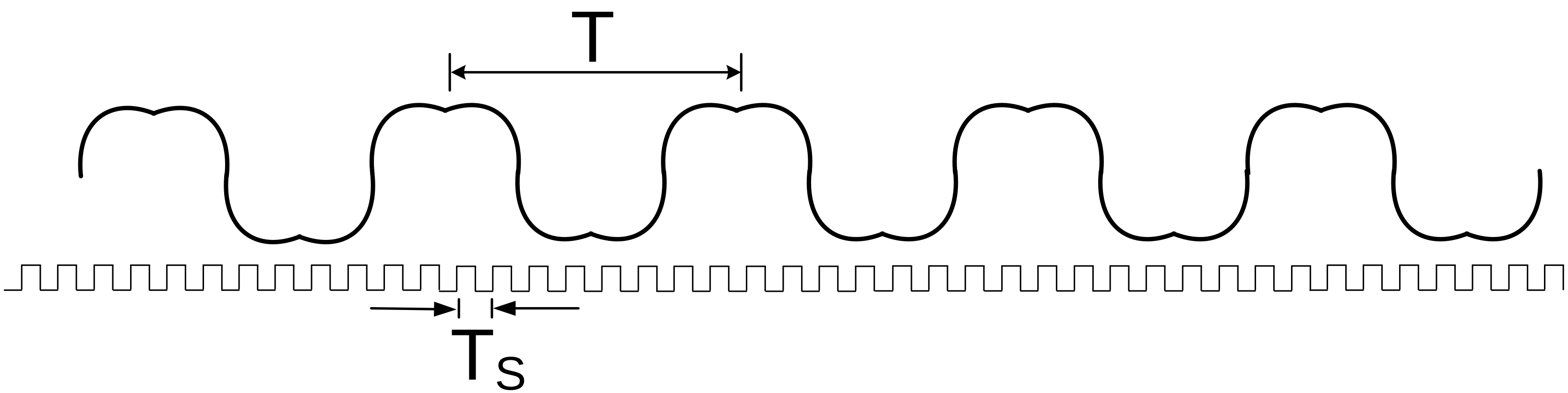
Consider a function f(t) that is periodic with period T

$$f(t) = A_0 + \sum_{k=1}^{\infty} A_k \sin(k\omega t + \theta_k) \qquad \omega = 2\pi f = \frac{2\pi}{T}$$

Band-limited Periodic Functions

Definition: A periodic function of frequency f is band limited to a frequency f_{max} if A_k=0 for all $k > \frac{f_{max}}{f}$

Distortion Analysis



NOTATION:

T: Period of Excitation
T_S: Sampling Period
N_P: Number of periods over which samples are taken
N: Total number of samples

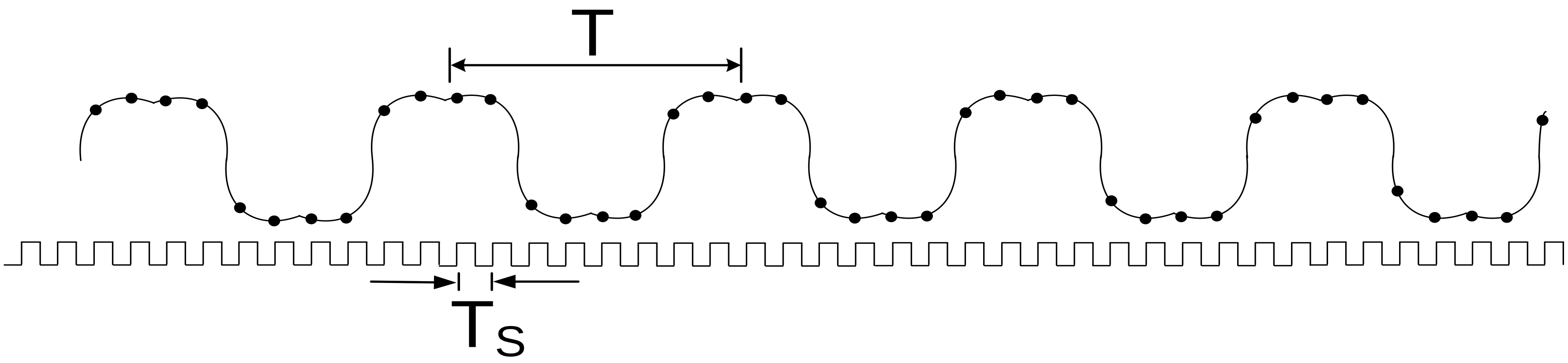
$$N_P = \frac{NT_S}{T}$$

Note: N_P is not an integer unless a specific relationship exists between N, T_S and T

$$h = \mathrm{Int}\left(\left[\frac{N}{2}-1\right]\frac{1}{N_P}\right)$$

Note: The function Int(x) is the integer part of x

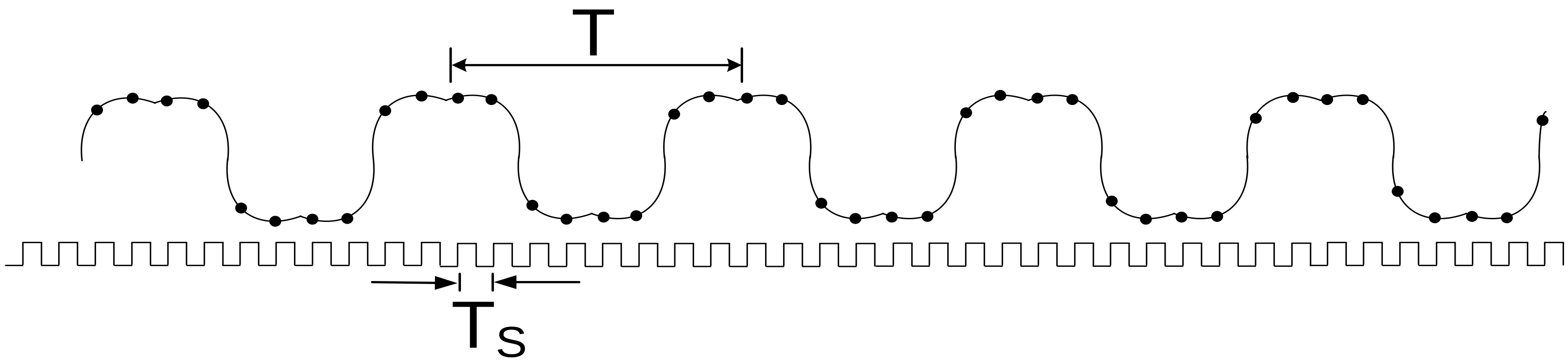
Distortion Analysis



THEOREM (conceptual) : If a band-limited periodic signal is sampled at a rate that exceeds the Nyquist rate, then the Fourier Series coefficients can be directly obtained from the DFT of a sampled sequence.

$$\left\langle x\left(kT_S\right)\right\rangle_{k=0}^{N-1} \Longleftrightarrow \left\langle X\left(k\right)\right\rangle_{k=0}^{N-1}$$

Distortion Analysis



THEOREM: Consider a periodic signal with period T=1/f and sampling period T_S=1/f_S. If N_P is an integer and x(t) is band limited to f_{MAX}, then

$$\left|A_m\right| = \frac{2}{N}\left|X\left(mN_P+1\right)\right| \qquad 0 \le m \le h-1$$

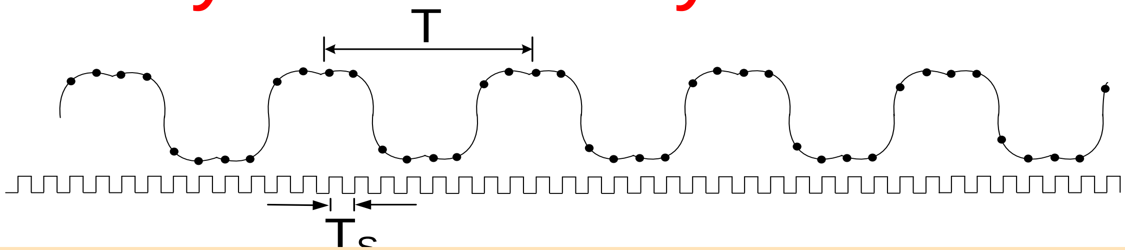
and $\quad X(k)=0 \quad$ for all k not defined above

where $\left\langle X(k)\right\rangle_{k=0}^{N-1}$ is the DFT of the sequence $\left\langle x(kT_S)\right\rangle_{k=0}^{N-1}$

N=number of samples, N_P is the number of periods, and h = $\mathrm{Int}\left(\frac{f_{MAX}}{f} - \frac{1}{N_P}\right)$

Key Theorem central to Spectral Analysis that is widely used !!! and often “abused”

Why is this a Key Theorem?



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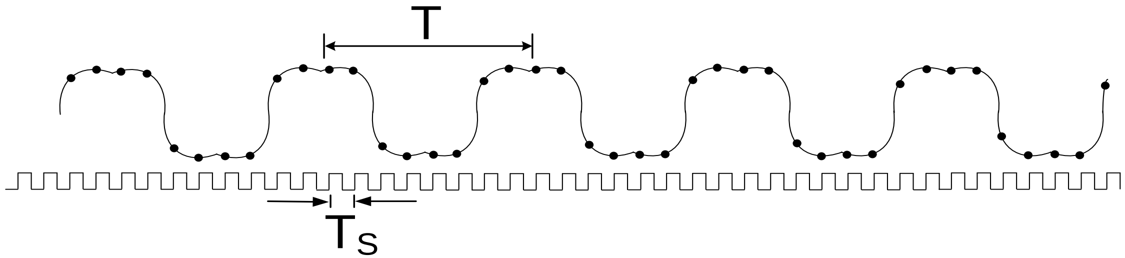
N=number of samples, N_P is the number of periods, and $h = \mathrm{Int}\left(\frac{f_{MAX}}{f} - \frac{1}{N_P}\right)$

- DFT requires dramatically less computation time than the integrals for obtaining Fourier Series coefficients
- Can easily determine the sampling rate (often termed the Nyquist rate) to satisfy the band limited part of the theorem

How is this theorem abused?



THEOREM: Consider a periodic signal with period T=1/f and sampling period $T_S=1/f_S$. If N_P is an integer and x(t) is band limited to f_{MAX}, then

$$|A_m| = \frac{2}{N}\left|X(mN_P+1)\right| \qquad\qquad 0 \le m \le h-1$$

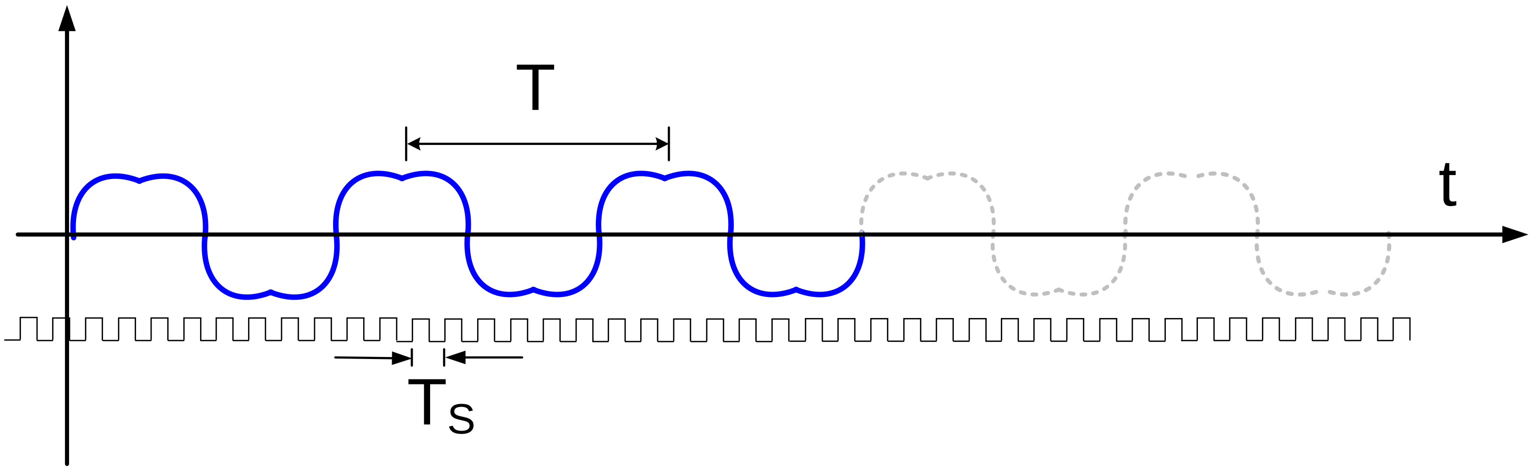
and $X(k)=0$ for all k not defined above

where $\langle X(k)\rangle_{k=0}^{N-1}$ is the DFT of the sequence $\langle x(kT_S)\rangle_{k=0}^{N-1}$

N=number of samples, N_P is the number of periods, and $h = \mathrm{Int}\left(\frac{f_{MAX}}{f} - \frac{1}{N_P}\right)$

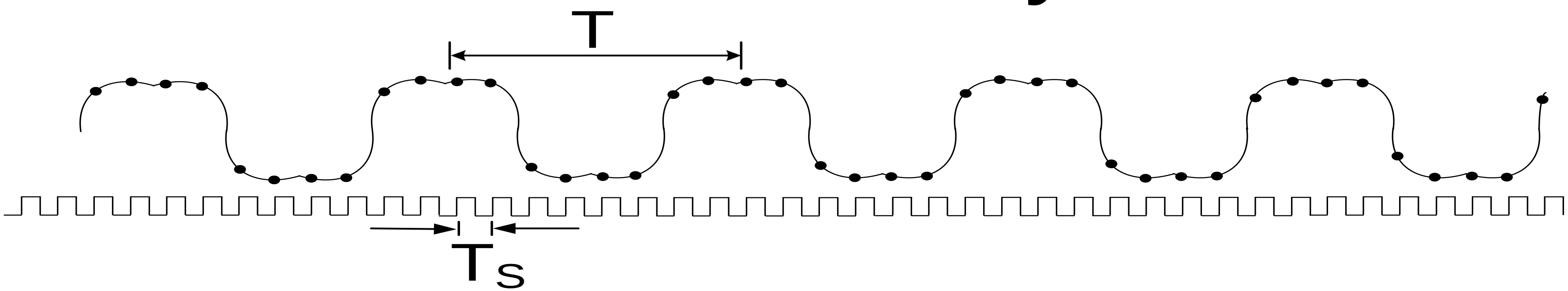
- Much evidence of engineers attempting to use the theorem when N_P is not an integer
- Challenging to have N_P an integer in practical applications
- Dramatic errors can result if there are not exactly an integer number of periods in the sampling window

3 Periods of Periodic Signal in Bold Blue

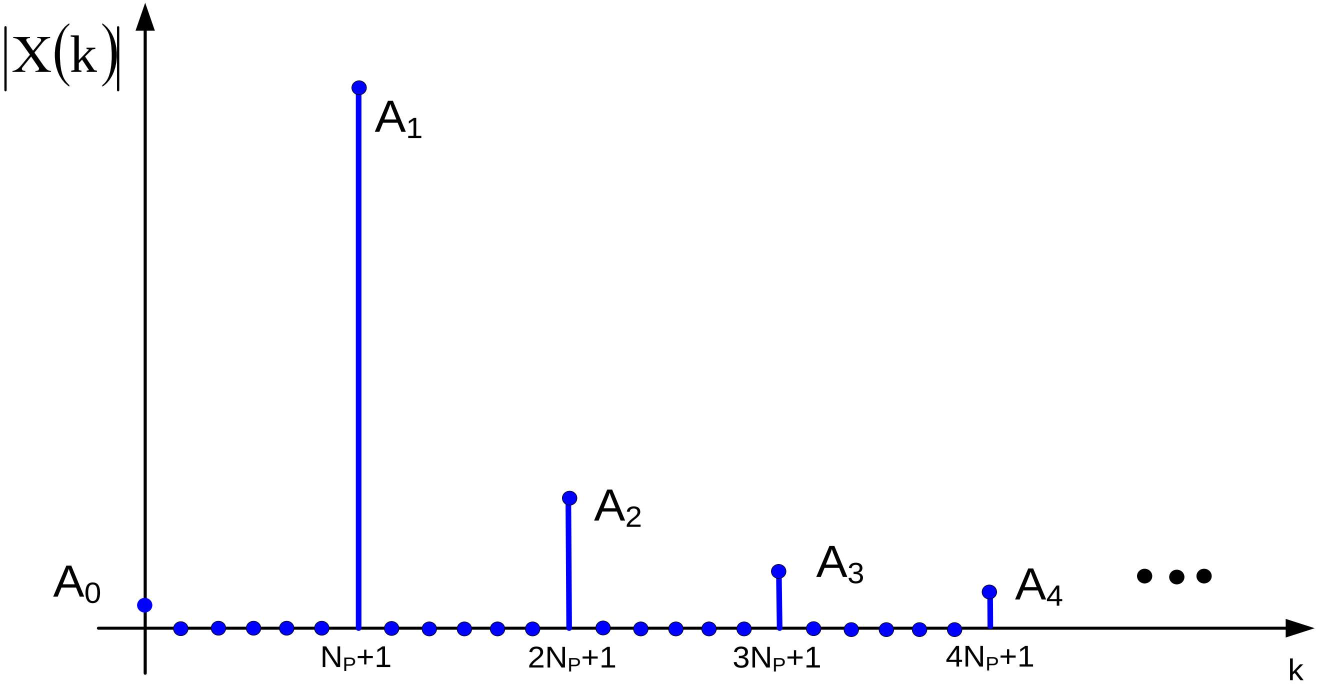


$$f_{SAMP} = f_{SIG} \frac{N}{N_P}$$

Distortion Analysis

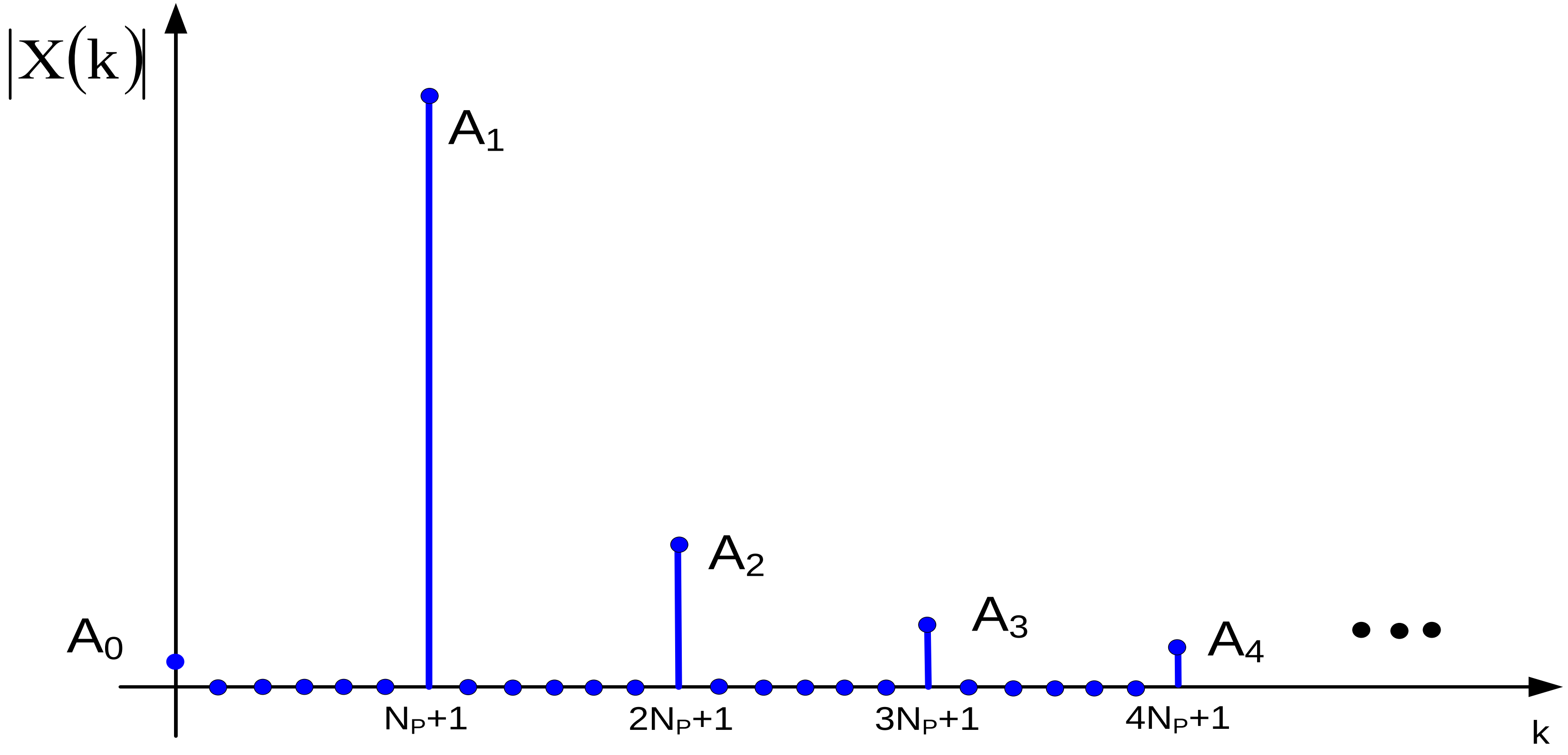


If the hypothesis of the theorem are satisfied, we thus have



Distortion Analysis

If the hypothesis of the theorem are satisfied, we thus have



FFT is a computationally efficient way of calculating the DFT, particularly when N is a power of 2

FFT Examples

Recall the theorem that provided for the relationship between the DFT terms and the Fourier Series Coefficients required

1. The sampling window be an integral number of periods

2. The input signal is band limited to f_{MAX}

Some notation and understanding related to Fourier Series, Discrete Fourier Series, Discrete Fourier Transform, Nyquist Rate, and Nyquist Frequency may be inconsistent from source to source, confusing, and not always correctly presented in all forums

From Wikipedia – March 30 2018

Discrete Fourier series

From Wikipedia, the free encyclopedia

A Fourier series is a representation of a function in terms of a summation of an infinite number of harmonically-related sinusoids with different amplitudes and phases. The amplitude and phase of a sinusoid can be combined into a single complex number, called a Fourier *coefficient*. The Fourier series is a periodic function. So it cannot represent any arbitrary function. It can represent either**:**

(a) a periodic function, or

(b) a function that is defined only over a finite-length interval; the values produced by the Fourier series outside the finite interval are irrelevant.

When the function being represented, whether finite-length or periodic, is discrete, the Fourier series coefficients are periodic, and can therefore be described by a finite set of complex numbers. That set is called a discrete Fourier transform (DFT), which is subsequently an overloaded term, because we don't know whether its (periodic) inverse transform is valid over a finite or an infinite interval. The term **discrete Fourier series (DFS)** is intended for use instead of *DFT* when the original function is periodic, defined over an infinite interval. *DFT* would then unambiguously imply only a transform whose inverse is valid over a finite interval. But we must again note that a Fourier series is a time-domain representation, not a frequency domain transform. So DFS is a potentially confusing substitute for DFT. A more technically valid description would be **DFS coefficients**.

Some notation and understanding related to Fourier Series, Discrete Fourier Series, Discrete Fourier Transform, Nyquist Rate, and Nyquist Frequency may be inconsistent and confusing

From Wikipedia – March 30 2018

Nyquist rate

From Wikipedia, the free encyclopedia

Not to be confused with Nyquist frequency.

This article **may be confusing or unclear to readers**. Please help us clarify the article. There might be a discussion about this on the talk page. *(January 2014) (Learn how and when to remove this template message)*

Nyquist frequency

From Wikipedia, the free encyclopedia

Not to be confused with Nyquist rate.

The **Nyquist frequency**, named after electronic engineer Harry Nyquist, is half of the sampling rate

The Nyquist frequency should not be confused with the *Nyquist rate*, which is the minimum sampling rate that satisfies the Nyquist sampling criterion for a given signal or family of signals. The Nyquist rate is twice the maximum component frequency of the function being sampled. For example, the *Nyquist rate* for the sinusoid at 0.6 f_s is 1.2 f_s, which means that at the f_s rate, it is being *undersampled*. Thus, *Nyquist rate* is a property of a continuous-time signal, whereas *Nyquist frequency* is a property of a discrete-time system.[4][5]

FFT Examples

Recall the theorem that provided for the relationship between the DFT terms and the Fourier Series Coefficients required

1. The sampling window be an integral number of periods
2. $N > \dfrac{2\, f_{max}}{f_{SIGNAL}} N_P$ (from $f_{MAX} \le \dfrac{f}{2} \bullet \left[\dfrac{N}{N_P}\right]$)

Considerations for Spectral Characterization

- Tool Validation
- FFT Length
- Importance of Satisfying Hypothesis
- Windowing

Considerations for Spectral Characterization

- Tool Validation (MATLAB)
- FFT Length
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FFT Examples

Recall the theorem that provided for the relationship between the DFT terms and the Fourier Series Coefficients required

1. The sampling window must be an integral number of periods
2. $N > \dfrac{2\, f_{max}}{f_{SIGNAL}} N_P$

Example

WLOG assume f_{SIG}=50Hz

$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t)$$

$$\omega = 2\pi f_{SIG}$$

$f_{MAX\text{-}ACT}$=100Hz

Consider N_P=20 N=512

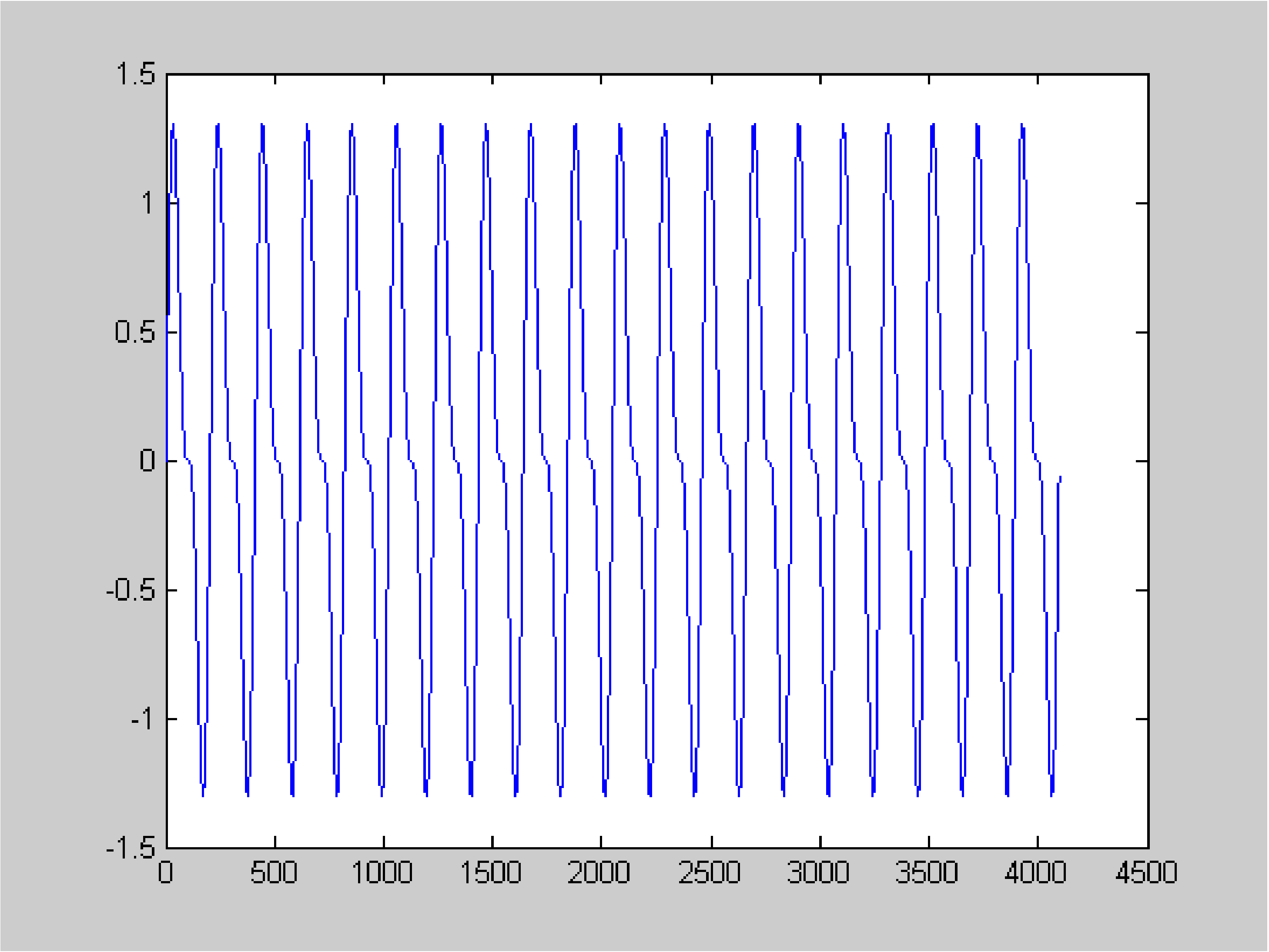
$$f_{MAX} = \frac{f_{SIG}}{2} \bullet \left[\frac{N}{N_P}\right] = \frac{50}{2} \bullet \frac{512}{20} = 640\,Hz \qquad f_{MAX\text{-}ACT} << f_{MAX}$$

$$f_{SAMPLE} = \frac{1}{T_{SAMPLE}} = \frac{1}{\left(\frac{N_P \bullet T_{SIG}}{N}\right)} = \left[\frac{N}{N_P}\right] f_{SIG} = 2 f_{MAX} = 1280\,Hz$$

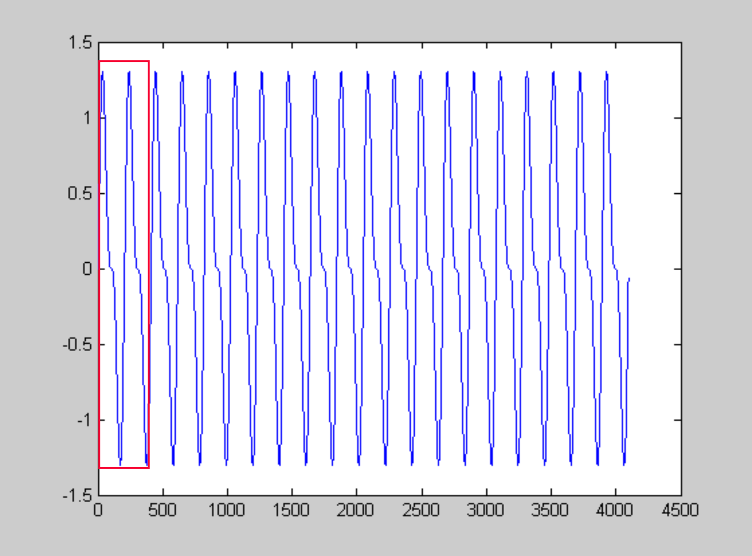
Recall $20\log_{10}(1.0)=0.0000000$

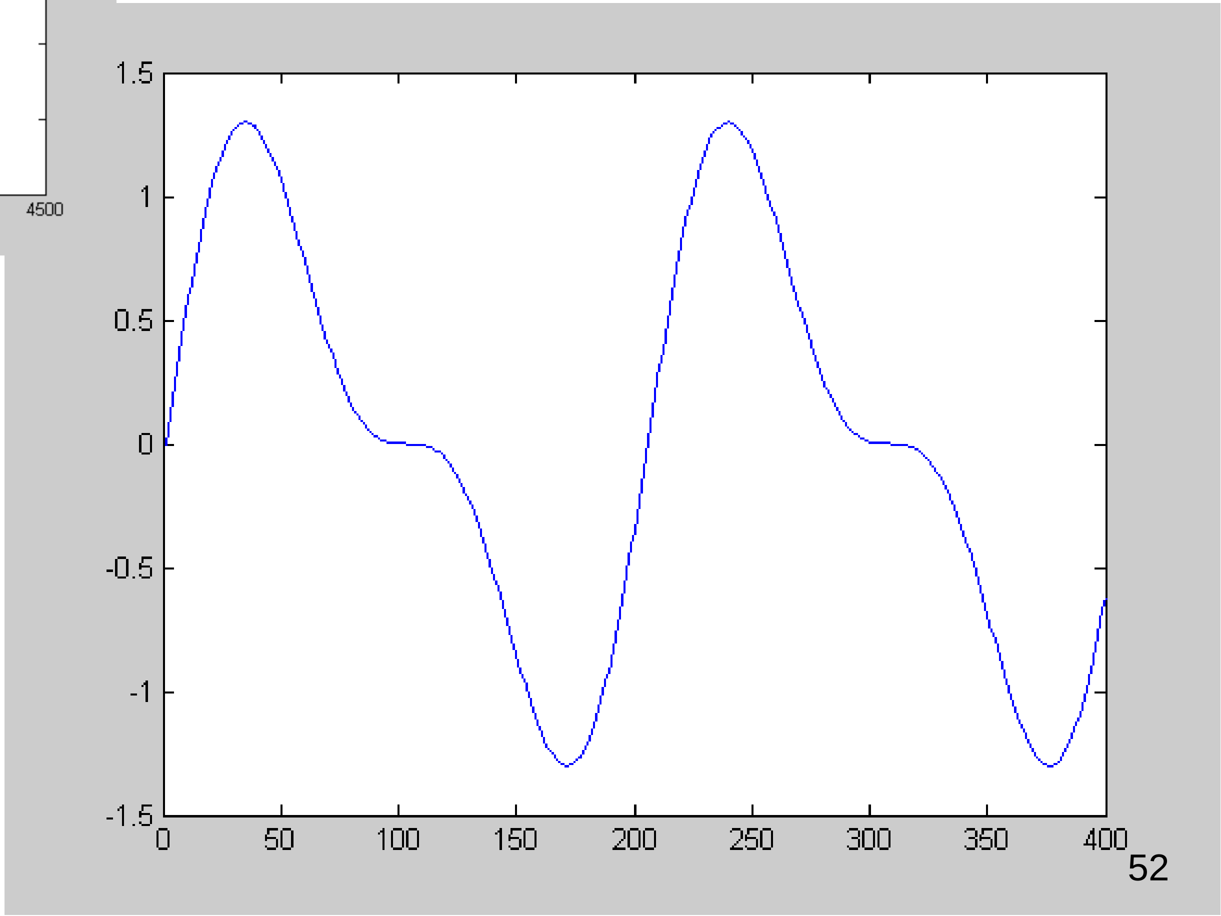
Recall $20\log_{10}(0.5)=-6.0205999$

Input Waveform

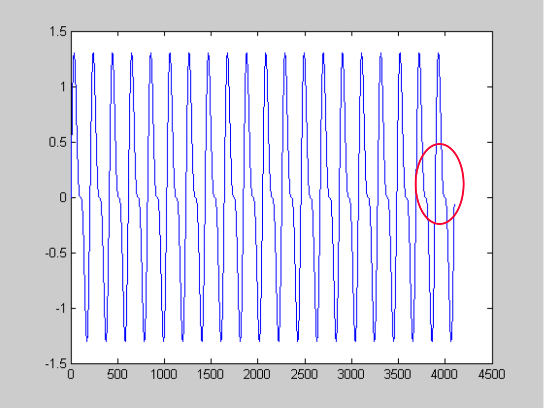


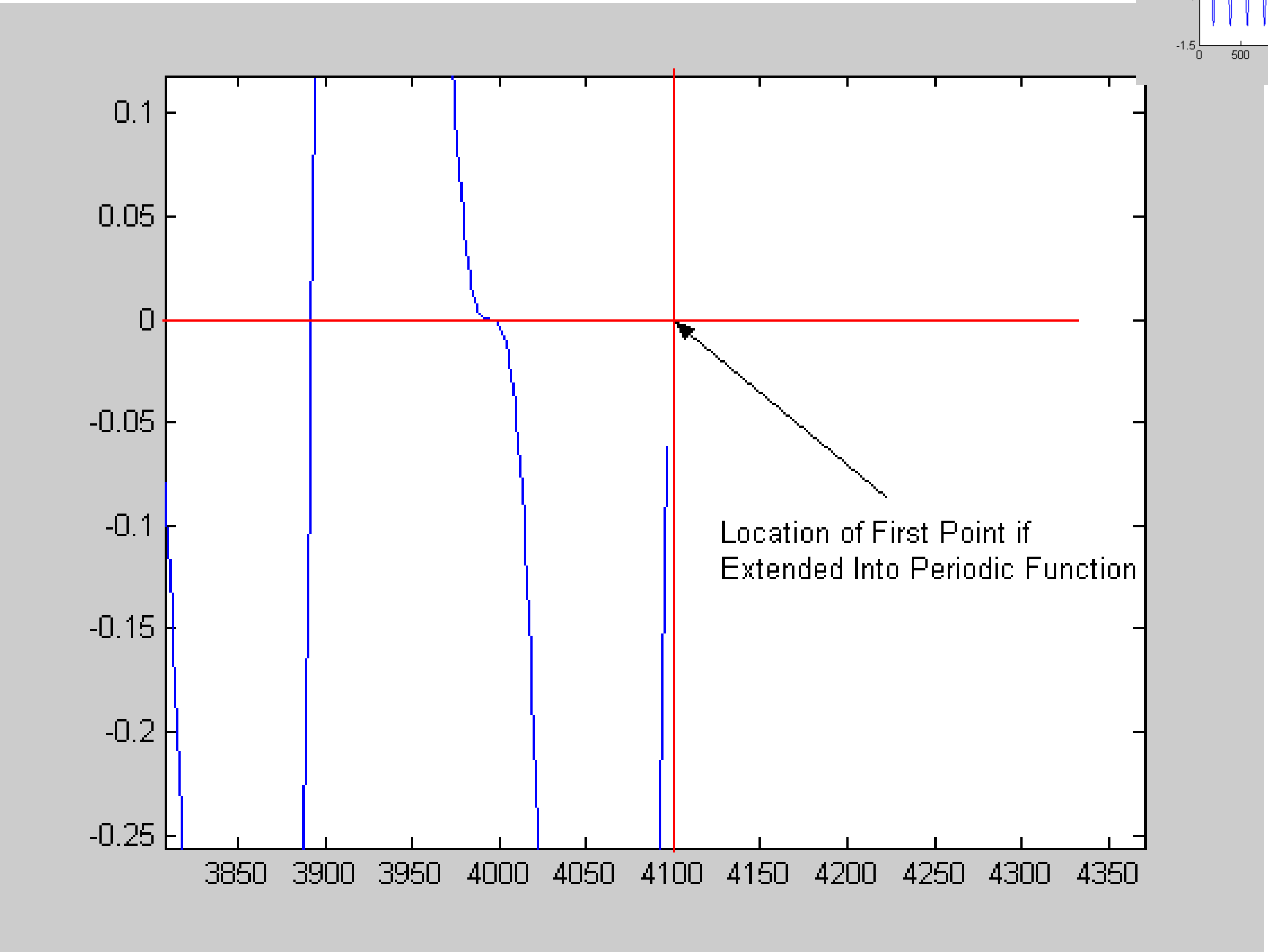
Input Waveform



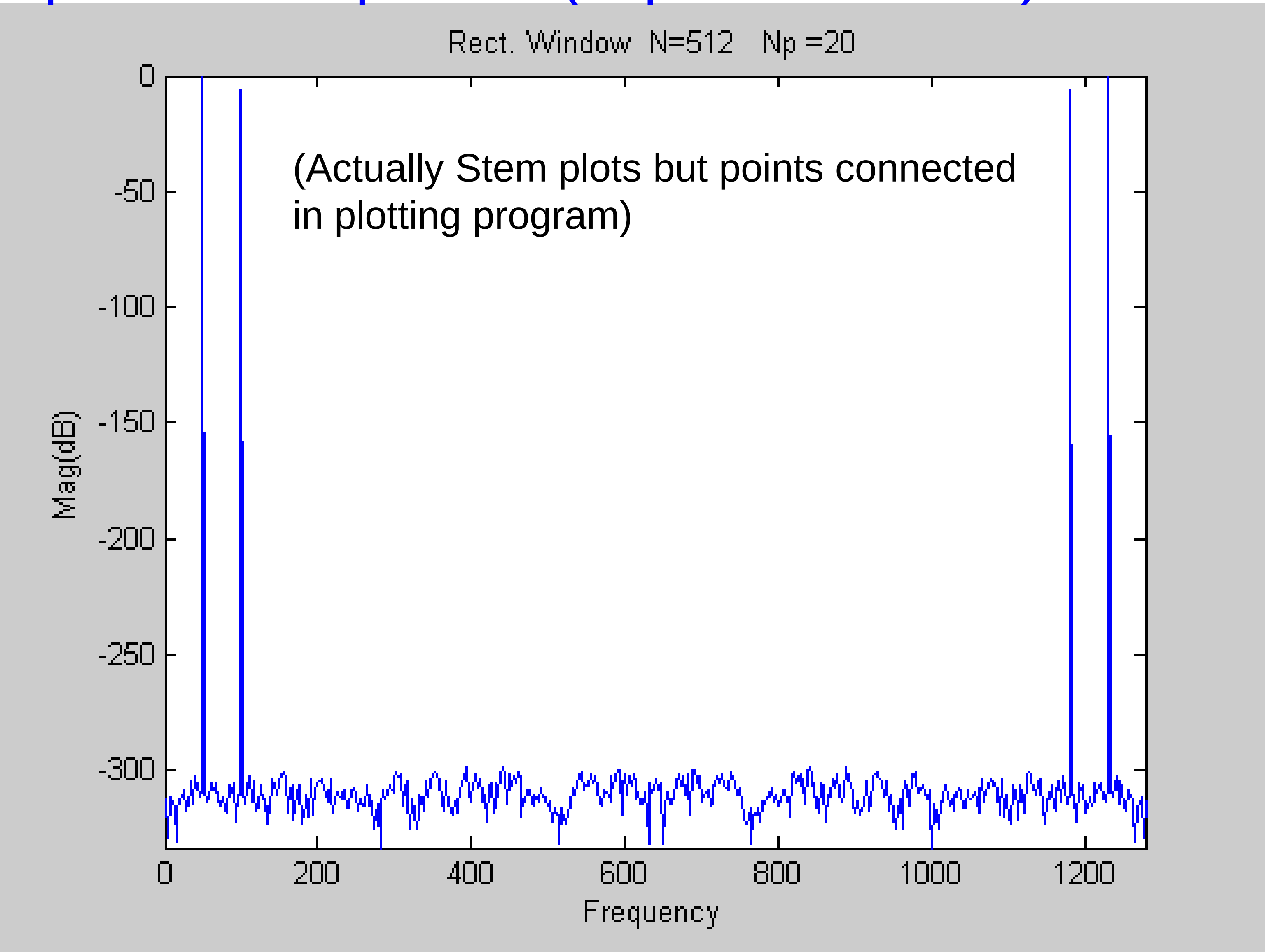


Input Waveform





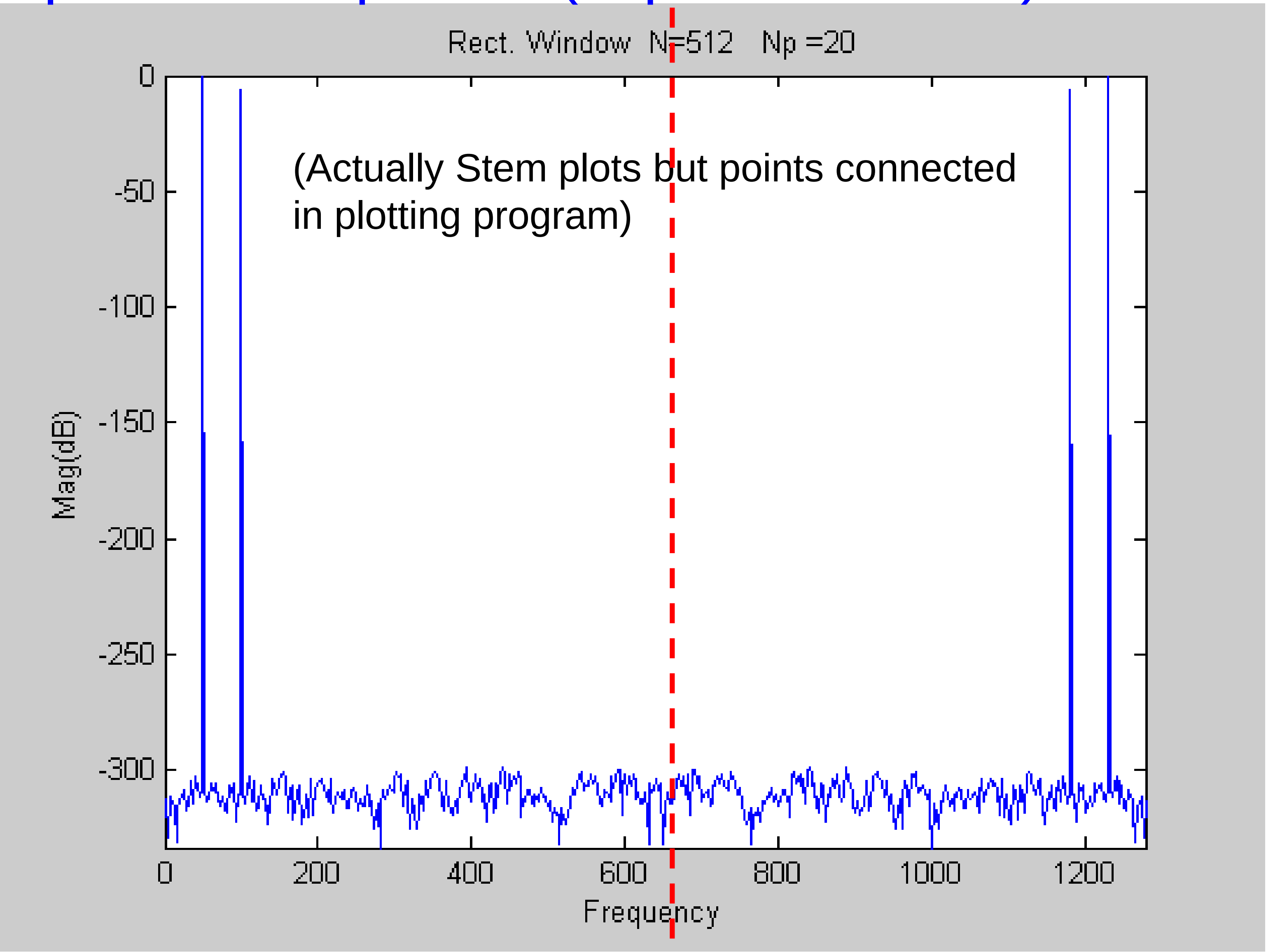
Spectral Response (expressed in dB)



(Horizontal axis is the "Index" axis but converted to frequency)

$$f_{AXIS} = f_{SIGNAL}\frac{n-1}{N_P}$$

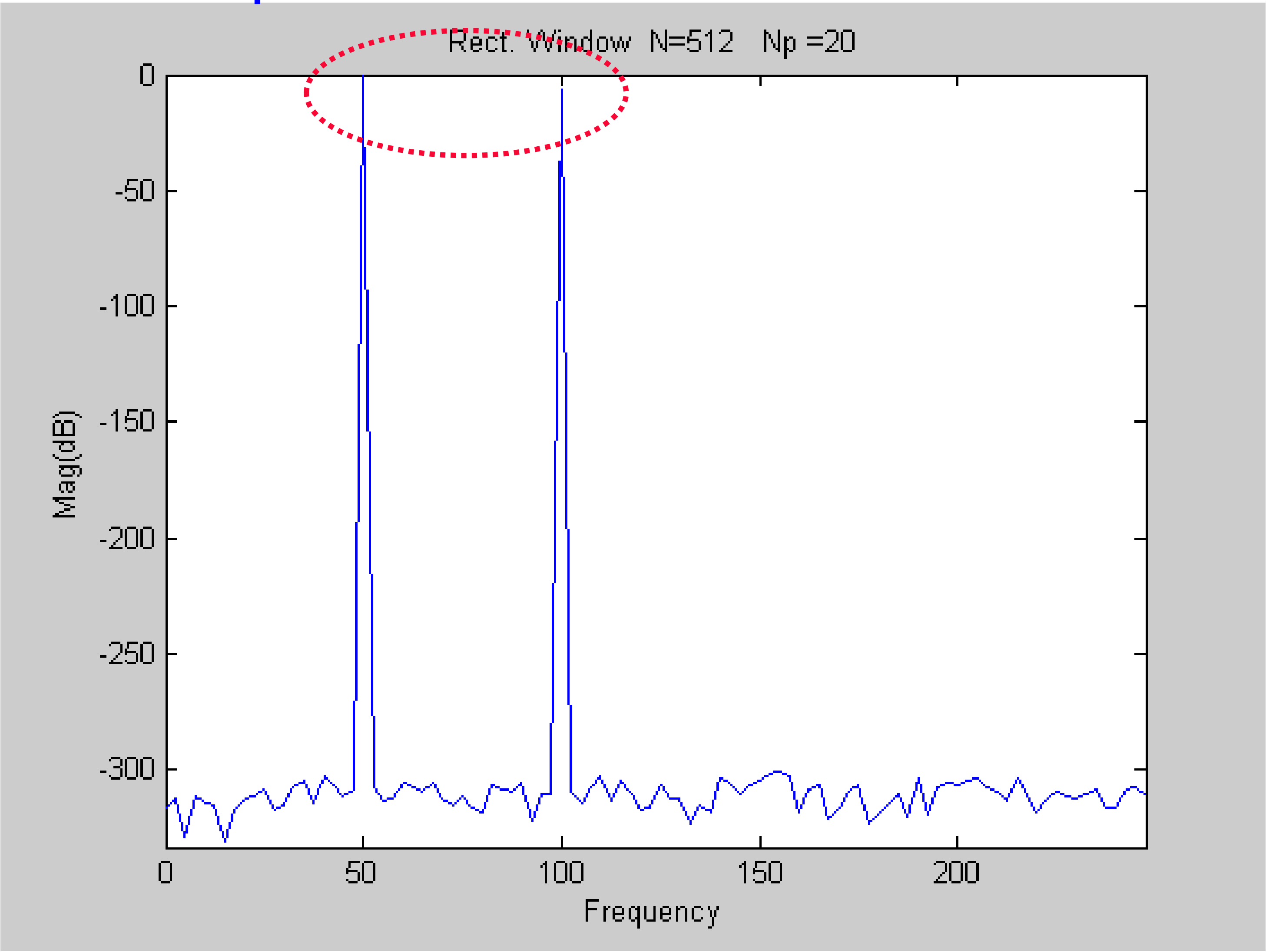
Spectral Response (expressed in dB)



Note Magnitude is Symmetric wrt f_{SAMPLE}

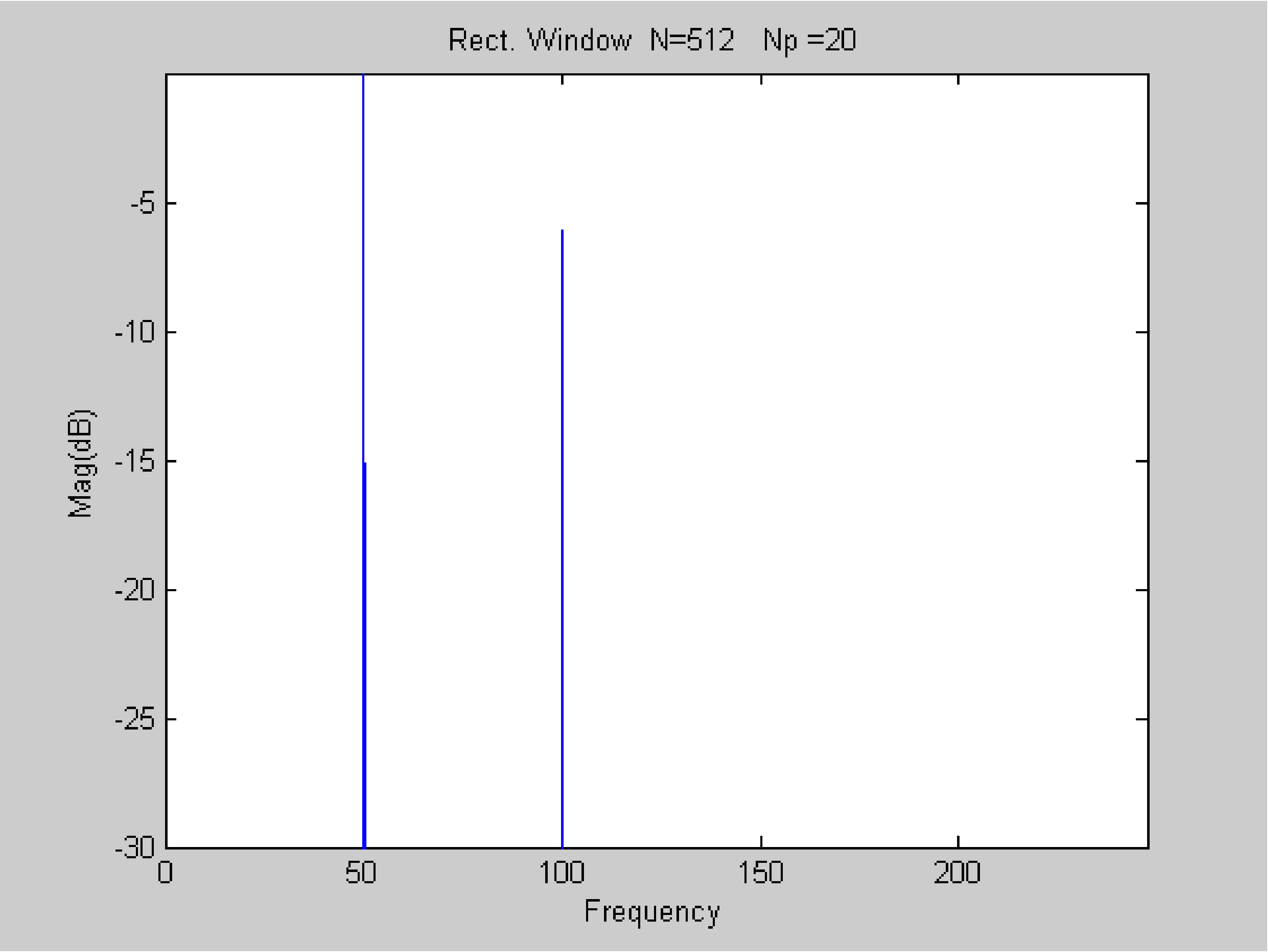
$$f_{AXIS} = f_{SIGNAL} \frac{n-1}{N_P}$$

Spectral Response



DFT Horizontal Axis Converter to Frequency : $f_{AXIS} = f_{SIGNAL} \dfrac{n-1}{N_P}$

Spectral Response



Fundamental will appear at position 1+Np = 21

Columns 1 through 5

-316.1458 -312.9517 -329.5203 -311.1473 -314.2615

Columns 6 through 10

-315.2584 -330.6258 -317.2896 -312.2316 -311.6335

Columns 11 through 15

-308.2339 -317.7064 -315.3135 -307.9349 -304.5641

Columns 16 through 20

-314.0088 -302.6391 -306.6650 -311.3733 -308.3689

Columns 21 through 25

$\boxed{-0.0000}$ -307.7012 -312.9902 -312.8737 -305.4320

Observe system noise floor due to both spectral limitations of signal generator and numerical limitations in FFT are below -300db

Second Harmonic at 1+2Np = 41

Columns 26 through 30

-307.8301 -309.0737 -305.8503 -312.2772 -315.7544

Columns 31 through 35

-311.9316 -316.0581 -318.3454 -306.4977 -308.6679

Columns 36 through 40

-309.9702 -305.9809 -322.1270 -310.6723 -310.3506

Columns 41 through 45

$\boxed{-6.0206}$ -309.6071 -314.1026 -307.6405 -302.9277

Columns 46 through 50

-313.0745 -304.2330 -310.8487 -317.7966 -316.3385

Third Harmonic at 1+3Np = 61

Columns 51 through 55

-307.0529 -312.7787 -312.9340 -323.2969 -314.9297

Columns 56 through 60

-318.7605 -303.5929 -305.2994 -310.6430 -306.7613

Columns 61 through 65

$\boxed{\text{-304.8298}}$ -301.4463 -301.1410 -303.1784 -317.8343

Columns 66 through 70

-308.6310 -307.0135 -321.6015 -316.6548 -309.8946

Columns 71 through 75

-306.3472 -323.0110 -319.3267 -314.7873 -310.4085

Fourth Harmonic at 1+4Np = 81

Columns 76 through 80

-319.8926 -303.3641 -319.6263 -307.6894 -305.1945

Columns 81 through 85

-306.8190 -304.8860 -303.6531 -307.2090 -309.8014

Columns 86 through 90

-313.4988 -303.4513 -310.4969 -317.9652 -312.5846

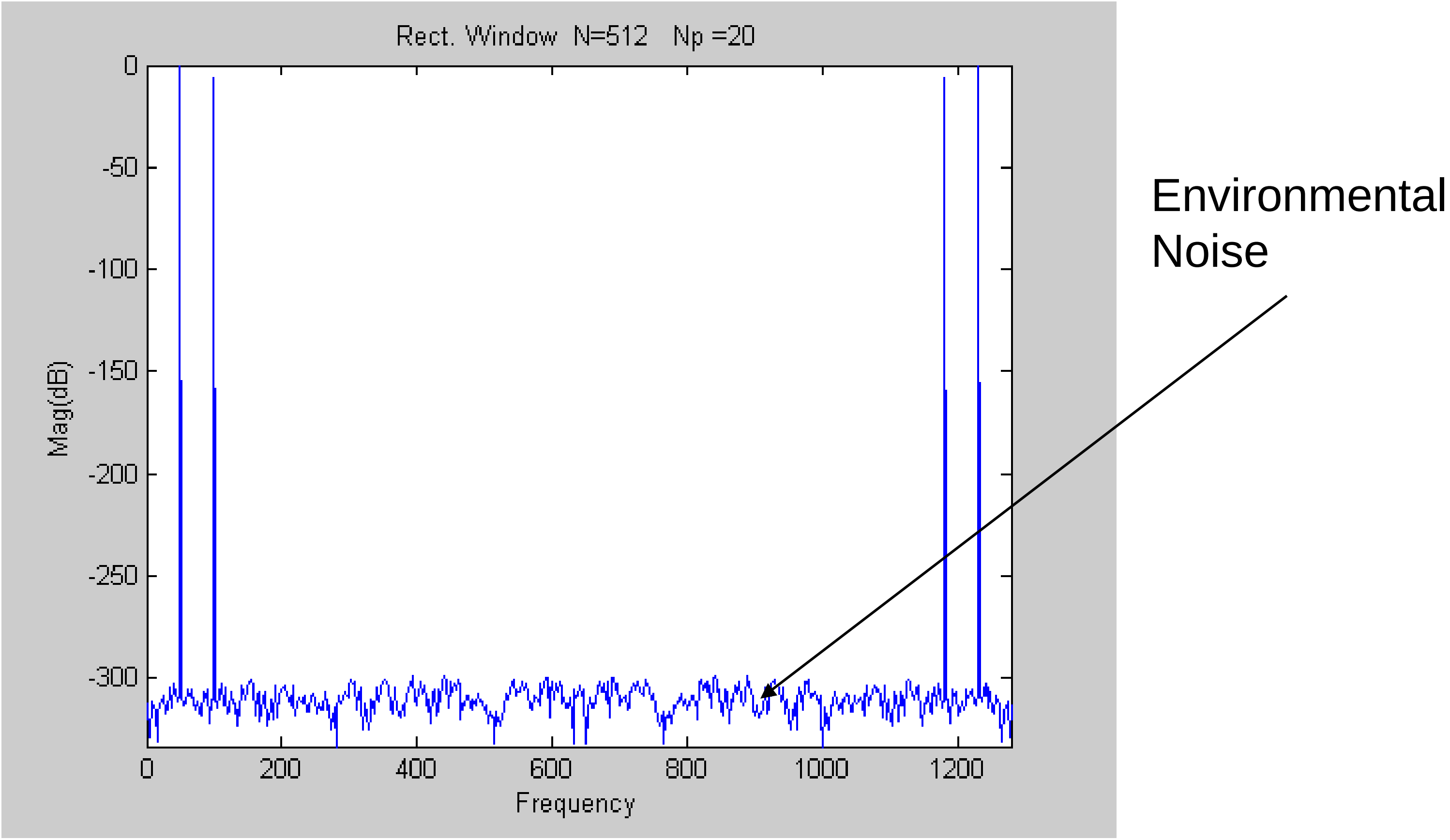
Columns 91 through 95

-309.8121 -311.6403 -312.8374 -310.5414 -308.7807

Columns 96 through 100

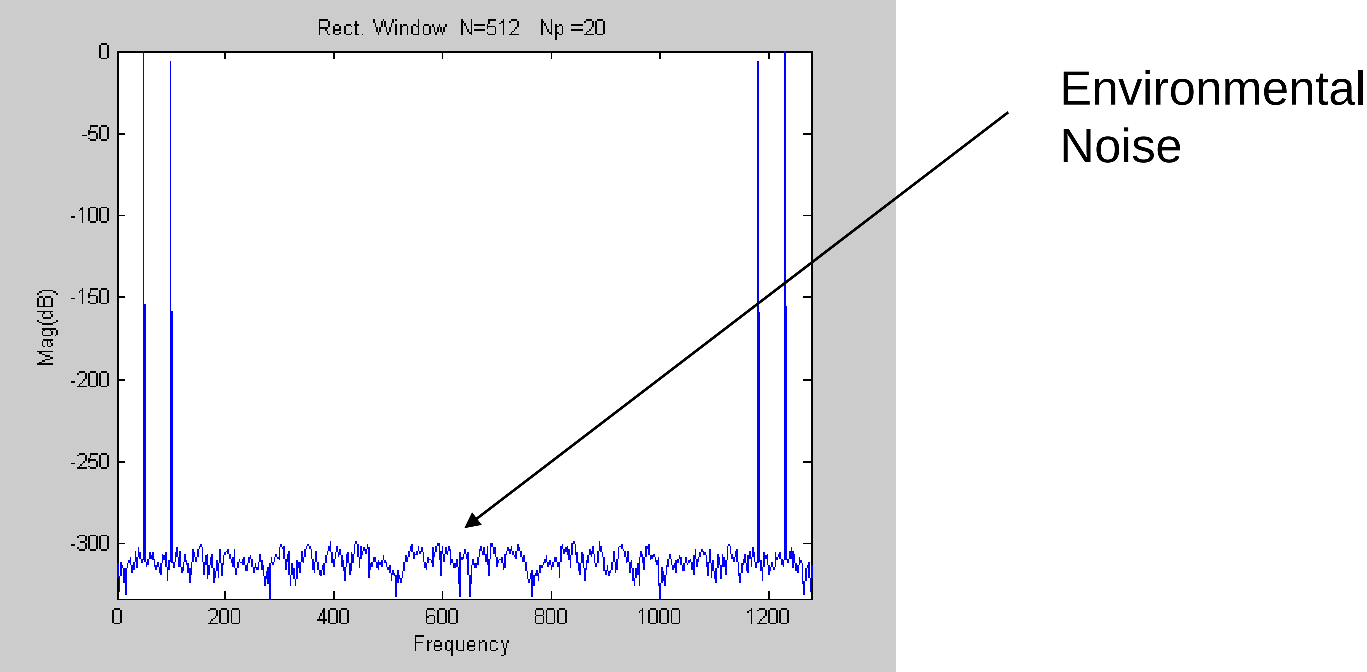
-316.7549 -316.3395 -308.4113 -307.3766 -311.0358

Question: How much noise is in the computational environment?



Is this due to quantization in the computational environment or to numerical rounding in the FFT?

Question: How much noise is in the computational environment?



Observation: This noise is nearly uniformly distributed

The level of this noise at each component is around -310dB

Stay Safe and Stay Healthy !

End of Lecture 27